ABSTRACT

At any manufacturing unit when a lot or batch of raw material is received, it goes through quality check. Doing quality check for each and every item of the lot can be both time consuming and costly. If in case the quality check is destructive (eg: testing light bulbs) by nature, then testing every item is not an idea to be pondered upon. But quality also shapes beliefs of the customers and hence can have a huge impact on sales. Therefore, the received lot cannot be passed without any quality check either. In order to solve the aforementioned problem, manufacturing and production industries employ the technique of acceptance sampling.

Typically an acceptance sampling plan (ASP) can be described as follows: Let us suppose that a shipment of raw materials of size N is received by a manufacturing unit. A sample of size n is drawn from a lot of raw materials received and some pre-specified quality characteristics are examined. The information obtained from the inspected sample is wielded to reach the conclusion on acceptance or rejection of the lot. ASP helps manufacturing houses in tackling the problem of selection of a lot or batch of raw materials or any other component units. The disparity between the required and the actual quality of supplied manufactured goods can be diminished considerably with ASP.

The lifetime of a product is one of the essential quality features of consumer durable products. In order to select a lot or a batch of raw material or finished product considering lifetime of the product as a quality attributed, life test sampling plans (LTSP) are used.

While testing the quality characteristic such as lifetime of a product we have to keep in mind that the quality attribute under study is not some instantaneously obtainable dimensional measurement. Due to the long duration and destructive nature of life testing experiments, LTSPs pose a two-fold challenge. Unlike other ASPs, it is not just required to decide what sample size is required to be tested, but also the stopping criterion of the life testing experiments. The stopping criterion is usually decided by the censoring schemes.

The two most commonly utilized censoring schemes are Type-I censoring (time-censoring) scheme and Type-II censoring (failure-censoring) scheme. In Type-I censoring, the test is aborted after a pre-decided time x_0; whereas in Type-II censoring, the termination of the test is subject to failure of a pre-fixed number of items r. In case of Type-I censoring, there is a possibility that very few or no failures are observed within the pre-decided time, whereas in Type-II censoring, since there was no cap on time, the experiment can take exceedingly high time. In order to address these drawbacks, the first hybrid censoring scheme which is popularly known in the literature as Type-I hybrid censoring scheme was first introduced by Epstein (1954) and can be considered as a mixture of Type-I and Type-II censoring schemes. In case of Type-I hybrid censoring scheme, the life test experiment is terminated at a random time $X^* = min\{X_{r:n}, x_0\}$ where, $X_{r:n}$, represents the time when r failures are observed.

The problem that arises out of Type-I hybrid censoring is that, there is a high chance of observing very few or no failures, like Type-I censoring scheme, if the mean lifetime of experimental units is greater than the censoring time, that is for highly reliable products. This lead to the introduction of Type-II hybrid censoring scheme by Childs et al (2003) which considered $X^* = max\{X_{r:n}, x_0\}$ as the termination time. But as in case of Type-II censoring, the test completion may take a very long time in Type-II hybrid censoring scheme. To overcome the limitations of both Type-I and Type-II hybrid censoring schemes, Chandrasekar et al. (2004) came up with Type-I and Type-II generalized hybrid censoring schemes. Under Type-I generalized hybrid censoring scheme (GHCS), the experiment is terminated at $X^* = min\{X_{r:n}, x_0\}$ only if l failures are observed before time x_0. If l failures occur after time x_0 has elapsed, then the experiment is terminated at x_0.

Due to the random nature of the lifetime of a product, different probability distributions are used to model them. LTSPs under different censoring schemes and different lifetime distributions have been studied extensively in the literature. Schneider (1989) developed failure censored life test plans for lognormal and Weibull distributions. Yeh (1994) used Bayesian approach to develop life test plans for products following exponentially distributed lifetimes under Type-I censoring setup. Balasooriya et al. (2000) determined life test plans for products following Weibull distributed lifetimes under Type-II progressive censoring. The method followed by Balasooriya et al. (2000) takes producer's and consumer's risks into account but no cost considerations are made in the design. A similar approach was followed by Bhattacharya et al. (2015), where life test plans were developed for Weibull distributed products under hybrid censoring setup. In addition to that they also followed variance minimization approach to obtain optimum life test plans. Chen et al. (2004a) proposed a general Bayesian framework for Weibull distributed product lifetimes with mixed censoring. Following a similar approach Chen et al. (2004b) presented life test plans for products with exponentially distributed lifetimes using random censoring schemes. Similarly, other scholars have approached the problem of developing LTSPs using different methodological approaches, but under most of these approaches cost considerations were seldom incorporated.

Since life testing involves as well as influences various costs, it is rational for a decision maker to consider various costs while designing LTSPs. For consumer durable products, warranty cost is one such cost component which cannot be ignored. Kwon (1996) was the first paper to include warranty cost as acceptance cost while designing optimal life testing plans for products with Weibull distributed lifetime under Type-II censoring setup employing a Bayesian approach. General rebate warranty policy was considered in the paper for calculation of warranty cost which is a combination of two most widely used elementary warranty policies for non-repairable products, free replacement warranty and pro-rata warranty. In free replacement warranty, a consumer can avail warranty services without any fee being incurred during the specified warranty period; whereas a pre-set proportion of the cost of repair is charged from a consumer on a pro-rata basis during the specified warranty period in case of pro-rata warranty. Following Kwon (1996); Huang et al (2008), Tsai et al (2008), and Hsieh and Lu (2013) included warranty cost in their studies under similar setup of Type-II censoring scheme. But to the best of our knowledge warranty cost has not been included for designing life testing plans in any other censoring setup. From a practical point of view, it can be witnessed that general rebate warranty as a warranty scheme and hybrid censoring as a life testing scheme are used frequently in automobile industry. The use of general rebate warranty can be seen widely in products such as tyres, car batteries and other automotive items. Hybrid censoring is also being used as a censoring scheme for automobile industry as can be observed from the real life data set used for analyzing the proposed life testing plans numerically and which is also reiterated by Blachre et al. (2015). This served as a practical motivation to combine the two for designing an appropriate testing plan.

The first two studies of this dissertation focuses on determining optimum life test sampling plans for products under warranty using Type-I hybrid censoring scheme and Type-I generalized hybrid censoring scheme under normal usage condition, which is the parlance of LTSP literature is also called usage stress setting. The product lifetimes in the aforementioned studies are assumed to follow Weibull lifetime distribution. Weibull distribution, which arises out of extreme value theory, is one of the most frequently used lifetime distributions in reliability engineering and survival analysis models, which is the reason why Weibull distribution is chosen for the purpose of this study. A constrained optimization approach to account for producer's and consumer's risk is inculcated in determining the life test sampling plans. A rigorous simulation study is conducted to validate the models for smaller samples by considering producer's risk. Since asymptotic distribution is used in deriving the lot acceptance criterion for the models, the simulation study verifies whether the life test sampling plans obtained works for smaller sample sizes. In order to study the sensitivity of the optimal solution due to mis-specification of parameter values and cost components, a well designed sensitivity analysis is incorporated using parameter estimates from real life Type-I hybrid censored data set for both the models. The results from the study highlight the importance of the parameters and assess the behavior of the optimal cost due to parameter changes. As warranty cost is included in the objective function to develop meaningful models for consumer durable products, the impact of its inclusion in the models is looked at. Insights on the behavior of optimal cost due to change in period of warranty is obtained for both the models. A significant change in optimal design is observed after inclusion of warranty cost in both the models.

While designing life testing plans for very high reliable products, it is impractical to use a simple LTSP since it can induce a lot of cost. In order to overcome this difficulty, accelerated life test plans (ALTSPs) are used. Shaked et al. (1979) introduced a nonparametric model of accelerated life testing. ALTSPs were developed by Yum and Kim (1990) for exponential distribution. Following Yum and Kim (1990); Hsiesh (1994) obtained ALTSP to minimize the total censoring number. Using Weibull distribution as lifetime distribution for the testing units Bai et al. (1995) developed failure-censored ALTSPs with equal expected test times at high and low test stresses. More recently, Seo et al (2009) developed ALTSPs for Weibull distributed lifetimes with a non-constant shape and scale parameters for both Type-I and Type-II censoring schemes. To the best of our knowledge, no study in the literature so far developed ALTSPs for Type-I hybrid censoring scheme for products sold under warranty. The third study in this dissertation develops ALTSPs for Type-I hybrid censoring scheme with non-constant scale parameter using a cost function approach for products sold under general rebate warranty scheme having Weibull lifetimes. As an alternative approach, accelerated life test sampling plans are also obtained using asymptotic variance minimization approach under the given setting. Evidence from the literature shows that ALTSPs using both the aforementioned approaches have not been studied for Type-I hybrid censoring scheme. A constrained optimization approach to account for producer's and consumer's risk is inculcated in determining the ALTSPs. A well designed sensitivity analysis is conducted by introducing a real-life failure dataset pertaining to locomotive controls. The results from the study highlight the importance of the parameters and assess the behavior of the optimal cost due to parameter changes. Insights on the behavior of optimal cost due to change in period of warranty is also highlighted.

An appropriate design of life test sampling plan is extremely important as some of the important decisions are dependent on that. If the design is not well thought through it may lead to poor quality of lot selection which in turn can shoot up other associated costs. The testing cost also can shoot up beyond control as a result of poor designing. For the products sold under warranty, an inappropriate LTSP can increase the number of warranty claims which will lead to rework, as a result of which cost in terms of efforts, time and money has to be borne by the company. Hence, for consumer durable products it is important to ensure that the cost due to warranty is induced while deciding on the design of life test sampling plans. Taking this practical approach in mind, this study focuses on the design of LTSPs for products sold under warranty incorporating the warranty costs. The LTSPs are designed using different censoring schemes under both usage stress and accelerated stress setup keeping in mind even the highly reliable products. In this sense the present work is an attempt to facilitate the decision making process in industries producing products under warranty.