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Abstract: Recently, Chowdhury et al. (2014a) proposed a single distribution-free Shewhart-type control chart based on the Cucconi (1968) test statistic for monitoring shift in the unknown location and scale parameters of a process distribution simultaneously. Several recent researches demonstrated that the CUSUM type charts perform better than the Shewhart-type charts under small and persistent shift. In the present work, we develop a phase II distribution-free CUSUM chart based on the Cucconi statistic, referred to as CUSUM-Cucconi (CC) chart. Nonparametric nature of the Cucconi statistic ensures that all the in control (IC) properties of the proposed chart remain invariant and known for all continuous process distributions. Control limits are tabulated for implementation of the chart. The IC and out of control (OOC) performance of the chart are thoroughly investigated in terms of the average, standard deviation, median and some percentiles of the corresponding run length distributions. A detailed comparison with the Shewhart-type Cucconi and Lepage charts as well as the CUSUM Lepage chart (as in Chowdhury et al. (2014b)) is presented. The proposed chart is illustrated with exchange rates data.

Keywords: Cucconi Statistic; Average Run Length; Upper Control Limit; CUSUM Cucconi Chart; Nonparametric; Monte-Carlo Simulation; Statistical Process Control.

MSC 2010 subject classification: 62L10, 62L05, 62G10, 62P30, 62P20

1. INTRODUCTION

Traditional statistical process control (SPC) charts use the normality assumption on the process distribution which is often found to be invalid in many practical situations. For instance, lifetimes of products can often be described by non-normal distributions viz. exponential, log-normal, Weibull etc. Even in many cases (see Qiu and Hawkins (2001, 2003), Qiu and Li (2011)), it is difficult to find a parametric distribution to model certain quality characteristic. In cases when the normality assumption is very hard to justify, several authors have pointed out that the actual false alarm rate of traditional control charts could be substantially larger or smaller than the assumed false alarm rate, resulting in either frequent disruptions of the production process or manufacturing of large number of defective products. One may see the works by Lucas and Crosier (1982), Rocke (1989), Hackl and Ledolter (1992), Amin et al. (1995) among others for further details.

To address these problems, host of researchers, see for example Bakir and Reynolds (1979), Bakir (2006), Chakraborti et al. (2011) advocated the use of distribution-free (nonparametric) control charts for monitoring either the process location or the scale parameter separately, in particular when the process distribution is unknown or known to be significantly different from normal. Sometimes functional form of the distribution is known but very complicated in nature. For a detailed discussion on nonparametric control charts, readers may see Chakraborti et al. (2001, 2007, 2011). Use of separate charts for different process parameters, specially, in nonparametric set-up, has certain practical limitations. Their simultaneous use often complicates inferential issues and interpretation apart from causing practical problems with regard to implementation. Several researchers, therefore, advocated use of a single chart for simultaneous monitoring of the two parameters instead of two separate charts for location and scale parameters. Interested readers may be referred to Cheng and Thaga (2006) that covers up the works in a detailed review of literature until 2005 and McCracken and Chakraborti (2013) for more recent advances. Despite being simpler in implementation, they may perform better than separate charts even in the parametric case as shown by McCraken et al. (2013).

Whereas a Shewhart control chart is designed to detect an immediate and substantial change in the process distribution, a cumulative sum (CUSUM) control chart is known to be advantageous to detect smaller, more persistent or cumulative shifts in the process distribution. Interested readers may see Reynolds et al. (1990), Yashchin (1992), Gan (1993, 2007), Chang and Gan (1995), Hawkins and Olwell (1998) and Goel (2011), Graham et al. (2014) and Chowdhury et al. (2014b) among others, for a detailed discussion on CUSUM control charting literature.

In the context of distribution-free CUSUM control charts, while Park and Reynolds (1987) developed nonparametric procedures for monitoring location parameter of a continuous process based on the linear placement statistic, McDonald (1990) considered a CUSUM procedure for individual observations based on the sequential ranks statistic. Bakir and Reynolds (1979) and Amin et al. (1995) proposed a nonparametric CUSUM chart based on the signed-rank and sign statistics, respectively. Run-length distribution of the CUSUM chart was discussed in detail by Jones et al. (2004). Li et al. (2010) considered the Wilcoxon rank sum test to detect step mean shifts through CUSUM and EWMA charts. Recently, Yang and Cheng (2011) and Mukherjee et al. (2013) developed a nonparametric CUSUM chart to detect the possible small shifts in process mean. Ross et al. (2011) discussed the problem of nonparametric monitoring of data streams for changes in location and scale and Ross and Adams (2012) considered two nonparametric control charts for detecting arbitrary distributional changes in the process. More details may be found in Chatterjee and Qiu (2009), Qiu and Li (2011) and Qiu (2013).

For monitoring both the location and scale parameters, Mukherjee and Chakraborti (2012) and Chowdhury et al. (2014b) considered nonparametric Shewhart-Lepage (SL) and CUSUM-Lepage (CL) charts based on the Lepage (1971) statistic. With the same objective, Chowdhury et al. (2014a) proposed a nonparametric Shewhart-Cucconi (SC) based on the Cucconi (1968) statistic. In particular, Chowdhury et al. (2014b) showed that the CL chart outperforms several other CUSUM charts, namely, the CUSUM chart based on Exceedance Statistic designed by Mukherjee et al. (2013) to monitor location shifts only; the CUSUM chart based on the Wilcoxon statistic designed by Li et al. (2010) that is used mainly for location shifts but also recommended for general shifts; as well as the CUSUM charts based on Kolmogorov-Smirnov statistic and Cramer-von-Mises statistic, for a wide class of location-scale models when the shift occurs in both the location and scale parameters. In this paper, we take the work another step forward and consider a nonparametric CUSUM chart based on the Cucconi (1968) statistic. We compare the proposed chart with the CL chart as well as the SL and SC charts and establish that the proposed chart is more effective than its competitors in various situations and is then preferable for overall monitoring.

The rest of the paper is organized as follows. Statistical framework and preliminaries along with a brief background on the Cucconi statistic are outlined in Section 2. Steps for implementation of the proposed CUSUM-Cucconi (CC) chart are introduced in Section 3 along with brief reference to the post-signal follow-up. Section 4 is devoted to the IC performance of the chart including run length properties and determination of the charting constant *H*. OOC performances along with a detailed comparison with the CL, SL and SC charts, are presented in Section 5. The charting procedure is illustrated in Section 6 with a new interesting financial data set. Section 7 concludes with a summary and directions for future research.

2. STATISTICAL FRAMEWORK AND PRELIMINARIES

Marozzi (2009) showed that the Cucconi test performs like or better than the more well-known Lepage test in many cases in the context of the testing of equality of both the location and scale parameters of two continuous distributions in the distribution-free framework. Many nonparametric tests for jointly testing location and scale parameters are based on the combination of two tests, one for location and one for scale, see the comparison study by Marozzi (2013). The most popular combination is the sum of the respective squared standardized test statistics, as used in the Lepage test. The Cucconi test statistic is not a combination of a test statistic for location and one for scale differences, but considers the squares of ranks and 'contrary ranks' making computations easier. Marozzi (2009) performed a detailed power simulation study including distributions of different shapes and showed that the Cucconi test maintains its size very close to the nominal level and is more powerful than the Lepage test in several situations. It was also seen that the presence of ties did not lower the performance of the Cucconi test contrary to the Lepage test. Motivated by these observations, we consider an adaptation of the Cucconi test to propose a CUSUM control chart for the joint monitoring of location and scale parameters of a continuous process.

Let $U_1, U_2, \dots, U_m$ and $V_1, V_2, \dots, V_n$ be independent random samples from two populations with cumulative distribution functions (cdf) $F(x)$ and $G(y) = F\left(\frac{x-\theta}{\delta}\right)$ with $\theta \in \Re, \delta > 0$, respectively, where F is some unknown continuous cdf. Here, the shift parameters θ and δ stand for unknown location and scale, respectively. Introduce an indicator variable I_k=0 or 1 as the k-th order statistic of the combined $N(= m + n)$ observations is a U or V. Define, the familiar Wilcoxon rank sum (WRS) statistic used to test the equality of the two location parameters as the sum of ranks of V_i in the combined sample of size N, given by $T_1 = \sum_{k=1}^{N} k I_k$.

Further consider the sum of the squares of the ranks of V_i's in the combined sample as: $S_1 = \sum_{k=1}^{N} k^2 I_k$. Suppose that the sum of anti-ranks of V_i and the sum of squares of anti-ranks of V_i in the combined sample are T_2 and S_2 respectively. It is easy to see from Chowdhury et al. (2014a) that the T_2 is given by: $T_2 = \sum_{k=1}^{N} (N+1-k) I_k = n(N+1) - T_1$, whereas S_2 is given by $S_2 = \sum_{k=1}^{N} (N+1-k)^2 I_k = n(N+1)^2 - 2(N+1)T_1 + S_1$.

In Phase II, the process is said to be in control if F=G, that is when $\theta = 0$ and $\delta = 1$. It is well known, see Gibbons and Chakraborti (2010), that $E(T_1 \mid IC) = \frac{1}{2} n(N+1)$ and $Var(T_1 \mid IC) = \frac{1}{12} mn(N+1)$. We can further observe from Marozzi (2009), Chowdhury et al. (2014a) that $E(S_1 \mid IC) = E(S_2 \mid IC) = \frac{1}{6} n(N+1)(2N+1)$ and $Var(S_1 \mid IC) = Var(S_2 \mid IC) =$ $\frac{1}{180} mn(N+1)(2N+1)(8N+11)$. Define the standardized S_1 and S_2 statistics as:

$$W = \frac{S_1 - E(S_1 \mid IC)}{\sqrt{Var(S_1 \mid IC)}} = \frac{6S_1 - n(N+1)(2N+1)}{\sqrt{\frac{1}{5} mn(N+1)(2N+1)(8N+11)}}$$

and

$$Z = \frac{S_2 - E(S_2 \mid IC)}{\sqrt{Var(S_2 \mid IC)}} = \frac{6S_2 - n(N+1)(2N+1)}{\sqrt{\frac{1}{5} mn(N+1)(2N+1)(8N+11)}}.$$

Note that in IC set-up, $E(W) = E(Z) = 0$ and $VAR(W) = VAR(Z) = 1$. Further, W and Z are negatively dependent with correlation coefficient taking values in the interval (-1,-7/8) and expressed as $Corr\,(W,Z) = \frac{2(N^2-4)}{(2N+1)(8N+11)} - 1 = \rho$. The minimum -1 occurs in the trivial situation where N=2, while the supremum is reached when N diverges to infinity with $\lim_{N\to\infty} \rho = -7/8$. When the process is OOC, that is when $\theta \neq 0$ and $\delta = 1$, or when $\theta = 0$ and $\delta \neq 1$, or when $\theta \neq 0$ and $\delta \neq 1$, one or both of $E(W)$ and $E(Z)$ are non-zero, the various situations are reported with more details by Marozzi (2009). In order to combine the information provided by both W and Z regarding the presence of a difference in location as well as in scale, Cucconi (1968) proposed the following rank based statistic:

$$C = \frac{W^2 + Z^2 - 2\rho WZ}{2(1-\rho^2)}.$$

It is important to note that the higher the deviation of θ and δ from 0 and 1 respectively, the larger is the value of C. Cucconi (1968) graphically showed that the acceptance region of the test based on C is an ellipse in the W-Z plane. Alternatively, C may be also interpreted using the concept of Mahalanobis distance. Note that this interpretation is novel in the sense that it was not visualized earlier by Cucconi (1968). We see that C may be seen as one half of the Mahalanobis distance between W and Z as:

$$C = \frac{1}{2}\begin{bmatrix} W \\ Z \end{bmatrix}^T \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}^{-1} \begin{bmatrix} W \\ Z \end{bmatrix}.$$

It is important to emphasize that the Mahalanobis distance takes appropriate account of the correlation ρ and is very different than the Euclidean distance between W and Z because ρ is very high (in absolute value).

Let C_j denote the C statistic computed on the j-th test sample. The upper one sided distribution free CUSUM-Cucconi chart to detect shift in location or/and scale is based on

$$CC_j = \max\left[0, CC_{j-1} + \left(C_j - \mu_{C_j}\right) - k\right] \text{ for } j = 1,2,..., \tag{2.1}$$

where the starting value $CC_0 = 0$, $\mu_{C_j} = E(C_j) = 1$ and $k \geq 0$ is called the reference value. Hence the proposed upper one-sided CUSUM-Cucconi plotting statistic is defined as

$$CC_j = \max\left[0, CC_{j-1} + \left(C_j - 1\right) - k\right] \text{ for } j=1,2,\ldots \tag{2.2}$$

Remark 2.1. It should be noted that an equivalent formulation of the C statistic is $\tilde{C} = W^2 + Z^2 - 2\rho WZ$. It is easy to see that $\tilde{C}$ is one to one and increasingly related with C because $2(1-\rho^2)$ is a positive constant. In that case, for $j = 1,2, \ldots, \ldots$ (2.2) may be rewritten as: $CC_j = \max\left[0, CC_{j-1} + \left(\tilde{C}_j - 2(1-\rho^2)\right) - k\right]$. However, keeping parity with Chowdhury et al. (2014a), we use the traditional C statistic in our computations and illustrations.

Remark 2.2. Even if the data comes from a continuous population there may be a few ties in practice due to rounding or truncation. Without ties the labeling of the samples (first or second) does not matter in computing C because $W^* = -W$ and $Z^* = -Z$ where W^* and Z^* denote respectively W and Z computed on the first sample. If ties are present, we obtain slightly different values of the Cucconi statistic by computing it on the first and second sample elements. In this case, we replace the C statistic with $(C+C^*)/2$ where C^* is the Cucconi statistic computed on the first sample elements. Such computational aspect in presence of ties was already noted by Cucconi (1968) which proposed a more complex solution.

3. PROPOSED CHARTING PROCEDURE

The proposed upper one-sided CUSUM-Cucconi (CC) control chart is constructed as follows.

Step-1: Collect and establish a reference sample $\boldsymbol{X_m} = (X_1, X_2, \ldots, X_m)$ of size m from an IC process. Establishment of a reference sample is itself an interesting problem; however, we are not considering the issue in this paper.

Step- 2: Sequentially observe the j-th phase II (test) sample $\boldsymbol{Y_{j;n}}=(Y_{j1}, Y_{j2},\ldots,Y_{jn})$ of size n, j=1,2,…

Step-3: Identify $\boldsymbol{X_m}$ as U and $\boldsymbol{Y_{j;n}}$ *as* V samples respectively and calculate W_j and Z_j for the j-th test sample following the equations in Section 2.

Step-4: Sequentially obtain the plotting statistic $CC_j = \max\left[0, CC_{j-1} + \left(C_j - 1\right) - k\right]$ for the j-th subgroup (j=1,2,…) of the CC chart starting with CC_0=0.

Step-5: Plot CC_j against an upper control limit (UCL) H. The lower control limit (LCL) is 0 by default as we have considered an upper one sided CUSUM chart.

Step-6: If CC_j exceeds H, the process is declared OOC at the j-th test sample. If not, the process is thought to be IC and testing continues to the next test sample.

When the process is declared OOC at the j-th test sample, Chowdhury et al. (2014a) proposed to compute the p-values for the Wilcoxon test for location and the Mood test for scale (see Gibbons and Chakraborti 2010) respectively applied to these two samples: $X = (X_1, X_2,\ldots, X_m)$ with the m Phase-I observations, and $Y_j = (Y_{j1}, Y_{j2},\ldots,Y_{jn})$ with the n observations from the j-th test sample. Denote the p-values as p_1 and p_2 respectively. If p_1 is very low but not p_2, a shift in only location is indicated. If p_1 is relatively high but p_2 is low, only a shift in scale is suspected. If both p-values are very low; a shift in both location and scale is declared. Note that it might happen that neither p_1 nor p_2 is very small even though CC_j is high, either because of an interaction between the location and scale changes or because of a false alarm. The same follow up procedure will work in the current context.

4. IC PROPERTIES OF THE CHART

4.1 Run Length Distribution

Mukherjee et al. (2013) mentioned that the run-length distribution of a CUSUM chart can be studied broadly via two approaches, namely, integral equation approach as in Page (1954) for continuous observations and Markov chain approach as proposed by Ewan and Kemp (1960) and further developed by Brook and Evans (1972). In the past four decades, host of researchers addressed this issue. Interested readers may see Barnard (1959), Bissell (1969), Champ and Rigdon (1991), Chao (2000), Crowder (1987a, b), Gan (1992), Khan (1978), Lucas and Crosier (1982), Reynold (1975), Robinson and Ho (1978), Saccucci and Lucas (1990), Vardeman and Ray (1985), Woodall (1983, 1984) and Waldmann (1986).

Unlike most of the previous work, we propose a Phase-II CUSUM chart and therefore, like in Mukherjee et al. (2013) first we need to approximate the conditional run length distribution of the CUSUM chart given $X_{(1)} < \cdots < X_{(m)}$. Let R denote the run length. Then the unconditional average run-length distribution is obtained by

$$\int_{-\infty}^{\infty} \dots \int_{-\infty}^{X_{(3)}} \int_{-\infty}^{X_{(2)}} E\big(R \mid X_{(1)} < \cdots < X_{(m)}\big)\ dF\ (X_{(1)})\ dF(\ X_{(2)}) \dots dF(\ X_{(m)})$$

$$= \ m! \int_0^1 \dots \int_0^{F^-(u_3)} \int_0^{F^-(u_2)} E\big(R \mid X_{(1)} < \cdots < X_{(m)}\big)\ du_1\ du_2 \dots du_m \qquad (4.1)$$

.

In order to implement the Markov chain approach to determine $E\big(R \mid X_{(1)} < \cdots < X_{(m)}\big)$, we consider, as in Fu et al. (2002), large but finite number of states, say ϑ (ϑ = 1,2,…) and $H = UCL$. Let $S_n(\vartheta)$ be a finite-state homogeneous Markov chain on the state space Ω with transition matrix Λ such that

(a) $\Omega = \{a_0, a_1, a_2, \dots, a_{\vartheta+1}\}$, where $a_0 = 0 < a_1 < \cdots < a_\vartheta < a_{\vartheta+1} = H$ where $a_i = (i - 0.5)d$, $i = 1,2,\dots,\vartheta$ with $d = \frac{h}{\vartheta + 1}$.

(b) $$\Lambda = \begin{pmatrix} T & \tilde{P} \\ \tilde{O} & 1 \end{pmatrix} = \begin{pmatrix} p_{00} & p_{01} \cdots\cdots p_{0\vartheta} & p_{0,\vartheta+1} \\ p_{10} & p_{11} \cdots\cdots p_{1\vartheta} & p_{1,\vartheta+1} \\ & \cdots\cdots\cdots & \\ & \cdots\cdots\cdots & \\ p_{\vartheta 0} & p_{\vartheta 1} \quad p_{\vartheta\vartheta} & p_{\vartheta,\vartheta+1} \\ 0 & 0 \ \cdots \ 0 & 1 \end{pmatrix}$$

where $p_{ij}$is the one-step transition probability from state *i* to state *j*; T is the transition probability sub-matrix with all the probabilities of going from one transient state to another; $\widetilde{P}$ is the column vector that contains all the probabilities of going from each transient state to the absorbing state; $\widetilde{O}$ is a null row vector that contains all the probabilities of going from the absorbing state to each transient state and the scalar value 1 is the probability of going from the absorbing state to the absorbing state. The elements of the sub-matrix T may be calculated from the conditional distribution of *Y* given $\boldsymbol{X_m}$. It is easy to see that, for $i = 0, 2, \dots, \vartheta$,

$$p_{i0} = P\left(CC_s = 0 \middle| CC_{s-1} = \frac{(i-0.5)h}{\vartheta + 1};\ \boldsymbol{X_m} = \boldsymbol{x_m}\right)$$

$$= \begin{cases} P\left(C_s \leq \dfrac{(k+1)(\vartheta + 1)}{(i-0.5)h} | \boldsymbol{X_m} = \boldsymbol{x_m}\right) & \text{if} \quad \dfrac{(k+1)(\vartheta + 1)}{(i-0.5)h} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.2}$$

Similarly for $i = 1, 2, \dots, \vartheta + 1$, and for $j = 2, 3, \dots, \vartheta + 1$, we have that

$$p_{ij} = P\left(CC_s = \frac{(j-0.5)h}{\vartheta + 1} \middle| CC_{s-1} = \frac{(i-0.5)h}{\vartheta + 1};\ \boldsymbol{X_m} = \boldsymbol{x_m}\right)$$

$$= \begin{cases} P\left(C_s \leq (1+k) + \dfrac{(j-i)h}{\vartheta + 1} | \boldsymbol{X_m} = \boldsymbol{x_m}\right) & \text{if} \quad (1+k) + \dfrac{(j-i)h}{\vartheta + 1} \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{4.3}$$

Define, $\mu_i = E(N_i | \boldsymbol{X_m} = \boldsymbol{x_m})$ as the conditional average run-length for $i = 0, 2, \dots, \vartheta$. Then, we have, from the properties of Markov chains,

$$\tilde{\mu} = (\mu_0, \mu_1, \dots, \mu_\vartheta)' = (I - T)^{-1}\tilde{1}.$$

It is easy to identify $\mu_0 = E(N_0 | \boldsymbol{X_m} = \boldsymbol{x_m}) = E(R | \boldsymbol{X_m} = \boldsymbol{x_m})$ for the chart with the starting value 0 and use it for to obtain unconditional average run-length using (4.1).

4.2. Determination of *H*

Note that determining ARL_0 by solving multiple integral equations using Markov Chain is not computationally straightforward. There are chances of very large computational error while approximating so many multiple integrals in order to estimate unconditional ARL_0. A Monte-Carlo simulation approach is therefore preferred in the present context. In order to determine the $UCL = H$ under a nominal ARL_0 of 250, 370 and 500, for different choices of m and n, we use a simple search algorithm. For a given m, n and k we determine ARL_0 for a sequence of values of H. A sequence of 16 equally spaced values of H that covers the range 250 to 500 for ARL_0 are considered. A predictive model for ARL_0 as a function of H based on smoothing spline is fitted using generalised cross validation technique for every given set of m, n and k. Given a triplet (m, n, k), we determine appropriate H for nominal ARL_0 of 250, 370 and 500 from the fitted model.

In the present article, we consider four different choices of m, the reference sample size. Those are $50, 100, 150$ and 300. We choose $n = 5$ and 11 as the representative of the size of test samples. For each of these 8 combinations of (m, n) three choices of reference value (k) are considered. Noting that the variance of the Cucconi statistic is 1 (Marozzi 2014), we consider $k = 0, 1.5$ and 3 in the line of Chowdhury et al. (2014b). More on choice of k is discussed later. We use the free software R (version: 2.15.3) to perform the entire Monte-Carlo experiment using 50,000 replicates. Our findings are presented in Table 1.

<<TABLE-1 TO BE PLACED HERE>>

4.3. IC performance of the chart

Traditionally, IC run length distribution of the CUSUM chart has a long right tail. The same phenomenon can be observed for the proposed CC chart. We see that irrespective of the choice of m, n and k, IC run length distribution of the proposed chart is positively skewed. Let us consider the Bowley skewness (also known as quartile skewness coefficient) of the IC run length distribution. We can see that for fixed n and k, the Bowley skewness decreases as m increases from 50 to 300. Similarly, for fixed m and n, the Bowley skewness decreases as k increases from 0 to 3. Initially, it decreases sharply with increase in k but rate of decrease is slow for $k > 1.5$.

However, for fixed m and k, the degree of skewness increases as n increases from 5 to 11. The rate of change in the skewness coefficient is relatively lower for large m and small k. In the expected line, the standard error of the ARL_0 decreases with increase in m for the proposed chart. However, for larger m (m=100, 150 and 300) the $SDRL_0$ decreases as n increases from 5 to 11 for k=0 but the $SDRL_0$ increases as n increases from 5 to 11 if k=1.5 or 3. For m=50 and k=0, the $SDRL_0$ values are almost the same for n=5 and 11, however, the $SDRL_0$ decreases as n increases from 5 to 11 when k=1.5 or 3. For fixed n and k, the median and 3-rd quartile values are slowly increasing as m increases while the 95-th percentile values decreases and therefore the degree of skewness of IC run length distribution is also reducing. In Table 2, we provide various percentiles of the run length distribution with the target ARL_0=500 for different combinations of m, n and k as used in Table 1. We also provide the corresponding values of the $SDRL_0$.

<<TABLE-2 TO BE PLACED HERE>>

We observe from Table 1 that the value of the upper control limit H of the CC chart decreases as k increases for fixed m, n and ARL_0 in the IC Set up. More importantly, Table 2 shows that the $SDRL_0$ for the CC chart decreases as k increases. Such phenomena can be observed for the CL chart discussed in Chowdhury et al. (2014b).

5. PERFORMANCE COMPARISONS

OOC performance of the proposed CC chart is compared with three other control charts designed for joint monitoring of location and scale parameters of an univariate process, namely, Shewhart-Lepage (CL) Chart of Mukherjee and Chakraborti (2012), Shewhart-Cucconi (SC) Chart by Chowdhury et al. (2014a) and CUSUM-Lepage (CL) Chart by Chowdhury et al. (2014b) for 48 pairs of location and scale shifts (θ, δ) values where θ = 0, 0.25, 0.5, 0.75, 1, 1.5, 2, 3 and δ = 0.5 (downward shift), 1 (no scale shift), 1.25, 1.5, 1.75, 2 (upward shift). For brevity, three population distributions, namely, the symmetric thin tailed normal, the symmetric heavy tailed Laplace and the skewed two-parameter exponential are considered for the purpose of comparison. We consider m = 100, 300 and n = 5. Results for normal distribution are presented in Table 3 and 4; the same for the Laplace distribution are displayed in Table 5 and 6. Finally, the findings of the two-parameter exponential distribution are placed in Table 7 and 8. The first row of each cell of Tables 3 to 8 is displaying the ARL and (SDRL) values while the second row is showing the 5-th, 25-th, 50-th, 75-th, 95-th percentiles in the ascending order.

We see, in general, that the OOC characteristics of the CC chart have some similarities in respect of its competitors, namely, the SL, SC and CL charts. Among the similarities, the most important feature is the nature of the OOC run length distribution. It has a long right tail for smaller shifts and is tending to degenerate for larger shifts. Secondly, the OOC ARL, SD and percentiles also decrease sharply with the upward shift in the location or/and scale and finally the CC chart, like its competitors, detects shift in the scale faster than that in the location.

<<TABLES-3 to 8 TO BE PLACED HERE>>

From Table 3 we see that for normal distribution the CC chart detects any type of shift (in location or/and in scale) of any magnitude (small, moderate or large) faster than the competitive charts except for the decreasing scale shift accompanied by moderate location shift (θ=1.5, δ=0.50), where the CL chart is slightly advantageous. For smaller shift the choice $k = 0$ is suitable while for larger shift $k = 1.5$ is preferable. The ARL bias (that is OOC ARL is greater than IC ARL, see Graham et al. (2014) for further details) is prominent for decreasing scale shift accompanied by small location shift when $k > 0$. From Table 4, we see that when m increases, the similar features are present in favour of the CC charts for location shift accompanied by no scale change or upward scale shift. However, there are slight dissimilarities as well. Note that, here $k = 1.5$ is preferable over a wider window of shifts and $k = 0$ is preferable only for small shift in both location and/or scale and downward scale shifts. Moreover, the CL chart is overall preferable for downward scale shift as mentioned earlier.

From Table 5, we observe that for the Laplace distribution, the rate of detection is slightly slower than in case of the normal. We further see that as in the thin tailed normal distribution, there is distinct benefit of the CC chart in detecting any type of location shift occurred in conjunction with upward or no scale shift for heavy tailed Laplace distribution. For downward scale shift, the CC chart is better when the scale shift is associated with small to moderate ($0.25 \leq \theta \leq 1.0$) location shift but otherwise, the CL chart is better. Here also, we observe the ARL bias for decreasing scale shift accompanied by small location shift when $k > 0$ is considered. We observe similar phenomenon from Table 6 where $m = 300$. For downward scale shift, the CC chart is better only when the scale shift is associated with moderate ($0.50 \leq \theta \leq 0.75$) location shift.

Further, in case of normal distribution (Table 4) for larger m, $k = 1.5$ is preferable over a wider window of shifts.

From Table 7 and 8 we see that for the skewed two-parameter exponential distribution the CC chart captures the shift in scale faster than the others when the location parameter remains in control. Interestingly, for shift in both location and scale, performance of the charts is mixed. The CC chart is also the best performer in detecting smaller shift in location accompanied by slight upward shift or no shift in scale. The SL chart performs slightly better than the CC chart for moderate to large shifts in location (usually for $\theta \geq 1$, except some cases with $m = 100$) when there is no scale shift or some upward scale shift. For very large shift in location, all the charts including the Shewhart type charts behave similarly. Performance of the charts in case of reduction in scale is somewhat different. For large shifts in location, all the charts behave similarly as expected whereas, for small to moderate changes in location, the CC chart with k=0 performs better than the others when m=100. On the other hand, the CL chart with k=0 detects shifts earlier than the other charts when m=300. This feature for downward scale shift is visible even for the symmetric distributions addressed earlier. There are no clear winner between the CL and the CC charts for downward scale shift, however, for all other situations, the CC chart is displaying better performance overall.

Remark 5.1. In connection to a two sample location-scale problem, Marozzi (2009 and 2013) established that when the ratio m/n of the two sample sizes does not exceed 3, the Cucconi test is generally markedly more powerful than the Lepage test under normal and light-tailed distributions whereas under heavy-tailed distributions the Lepage test is slightly more powerful than the Cucconi test. The comparison between the CUSUM charts showed very interesting results:

considering only the smaller shifts (since larger shifts are detected very similarly by both charts), the CC chart performs better than the CL chart also for the Laplace distribution. For the normal distribution, without surprise the CC chart performs better than the CL chart. The results on the CL chart appear to contradict Marozzi (2009 and 2013) results, however, previous results refer to a typical two sample problem where the sample sizes are small and not too unbalanced whereas in statistical process monitoring the sample sizes are very unbalanced with the ratio m/n that may even exceed 50 or 100.

Remark 5.2. As regard to choice of k, in general, we can conclude that the detection of small to moderate shifts in location or/and scale is faster for the normal, Laplace and exponential distribution when k=0 including the case of reduction in scale. For any large shift, k=1.5 is recommended.

6. ILLUSTRATIVE EXAMPLES

Effectiveness and usefulness of the CC chart are established in this section with the help of data on daily exchange rates between the Indian Rupee (INR) and Euro currency between April 1, 2012 to March 31, 2014 obtained from the website of the Reserve Bank of India (RBI), the highest banking regulatory authority in India. The lower control limit of the CC chart is 0 by default. The detail analysis is shown in the following subsections.

6.1. Analysis of the Exchange Rates data

The second data set consists of daily exchange rates between the Indian rupee (INR) and Euro currency between April 1, 2012 and March 31, 2014. A total of 485 data are extracted from *the*

RBI website. In this context, it is worth mentioning that Qiu and Li (2014) among others also considered CUSUM charts for monitoring financial data. The authors illustrated CUSUM charts based on daily exchange rates between the Korean Won and US dollar.

An Autoregressive Integrated Moving Average (ARIMA) model is fit to our data set using the in-build "arima" of R. 2.15.0 software. ARIMA (5, 1, 4) model gives a reasonably good fit to the data with the fitted model as:

	AR1	AR2	AR3	AR4	AR5	MA1	MA2	MA3	MA4
Coefficients	0.4689	0.5899	0.3401	0.5671	0.177	0.5393	0.6454	0.231	0.7697
Std Error	0.1013	0.125	0.1064	0.1011	0.0507	0.0961	0.1117	0.1007	0.0858

For details on the ARIMA model, interested readers may see Brockwell and Davis (1996), Durbin and Koopman (2001) and Gardner et al. (1980) among others. It is to be noted that the plotting statistics for the CC chart is based on the residuals data set. Moreover, testing procedures for randomness and normality of the IC data set are discussed next in detail. The MLE of the innovations variance is estimated as 0.2512 whereas the log-likelihood and AIC are respectively -352.84 and 725.68. Details of the computational schemes may be found in R documentation. Mukherjee (2009, 2013) addressed in detail the notion of monitoring the residuals of a time series which is often an independent sequence of observations. We consider the residuals here and apply the Box–Pierce and the Box–Ljung test for randomness on the residual observations. We see that the observed *p*-value for the Box–Pierce and the Box–Ljung tests are respectively 0.9544 and 0.9543. A two sided run test for randomness on the residuals gives the *p*-value as 0.9637. All these tests suggest that the residuals may be safely taken to be random resulting in applicability of the present control charting procedure.

Next, we consider the first 32 weeks data of size 150, starting from Monday, April 2, 2012 to Friday, November 9, 2012 as an IC data set. Therefore, for this example m=150. Then, we may easily consider each successive block of 5 residuals as test samples implying that we have 67 test samples of size n=5 each. Also note that the one-sample Kolmogorov-Smirnov test for normality on the first 150 residuals yields a p-value of order 10^{-8} and that the Jarque-Bera normality test p-value is 0.00188. The Shapiro-Wilk normality test gives little higher p-value but still as low as 0.01198. The D'Agostino omnibus normality test also returns a p-value of 0.0035. In particular, the skewness and kurtosis tests exhibit p-values as 0.006843 and 0.04504 respectively. Therefore, from all these test procedures along with the tests for randomness, it is very much evident that the reference sample of size 150 is a random sample from a non-normal population. A nonparametric control chart for monitoring any shift in location-scale in this financial time series process could be easily carried out by the proposed CC chart. For a target ARL_0 of 500, for m = 150, n = 5 and for k = 0, 1.5 and 3, H values can be easily obtained from Table 1. The plotting statistics for 67 test samples are not shown for brevity and can be produced on request. The CC charts on exchange data set are shown in Figure 2.

Figure 2 reveals that the CC chart with $k = 0$ exhibits the first OOC signal at the 29th test sample and continues to show the same signal afterwards. This indicates that the exchange rates between INR and Euro currency started becoming instable from middle of June-2013, precisely, June 12, 2013. The cases of $k = 1.5$ and $k = 3$ show a different phenomenon as opposed to the case of k=0, except the first change point in terms of attaining instability from stability where both the CC charts demonstrate first OOC signal at the 28-th test sample (from June 5, 2013) very close to the case of $k = 0$. Unlike the case of $k = 0$, here the signal runs only up to the 47-th sample excluding the 36-th and 37-th test samples. Finally, from the 48-th test sample onwards there is no signal. The CC chart with k=3 shows the OOC signals on the 28-th and 29-th samples and then from the 39-th test sample onwards to the 42-nd test sample and there is no signal from the 43-rd test sample onwards. Clearly, large shifts are detected by all three charts and moderate shifts are captured by the charts with $k = 0$ and $k = 1.5$. Further small shifts are only detected by the chart with $k = 0$.

In general, we can say that the Euro currency exchange rates as compared to INR exhibit hyperinflation from the beginning of June, 2013. Although, the pattern sustains for k=0, the INR currency seems to recover the loss from around September, October, 2013 according to the CC chart with k=1.5 and k=3. The follow-up analysis suggests that the aspect most responsible for OOC is the scale one, in particular from the 28-th test sample.

7. SUMMARY AND CONCLUSIONS

The article is comprised of a new phase II distribution-free CUSUM chart based on the Cucconi statistic. All in control properties of the proposed chart remain invariant and known for all univariate continuous process distributions because it is a nonparametric chart. Control limits for some representative values of the reference sample size, the test sample size and the reference value are tabulated for practical implementation. The IC and OOC performance properties of the chart have been investigated through simulations in terms of various run length characteristics and compared with the Shewhart-type Cucconi and Lepage charts as well as the CUSUM Lepage chart. The CC chart is displaying markedly better performance over the other charts in case of both symmetric and skewed distributions in detecting shifts of various magnitudes in location and/or in scale.

As a possible future work, one may consider the construction of an EWMA chart based on the Cucconi Statistic. Further, implementation of the proposed charts using the notion of median run length as in Graham et al. (2014) may be another interesting future research problem. In this context, it is worth mentioning that construction of a suitable nonparametric self-starting chart for joint monitoring of the location-scale family is also long overdue.

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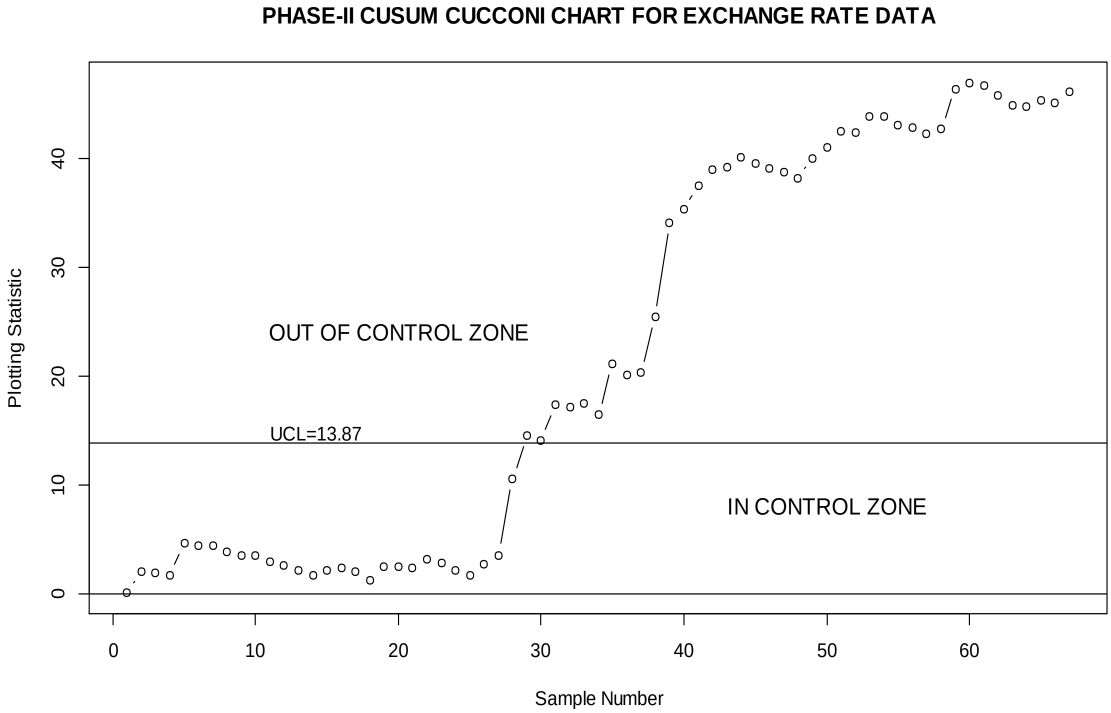
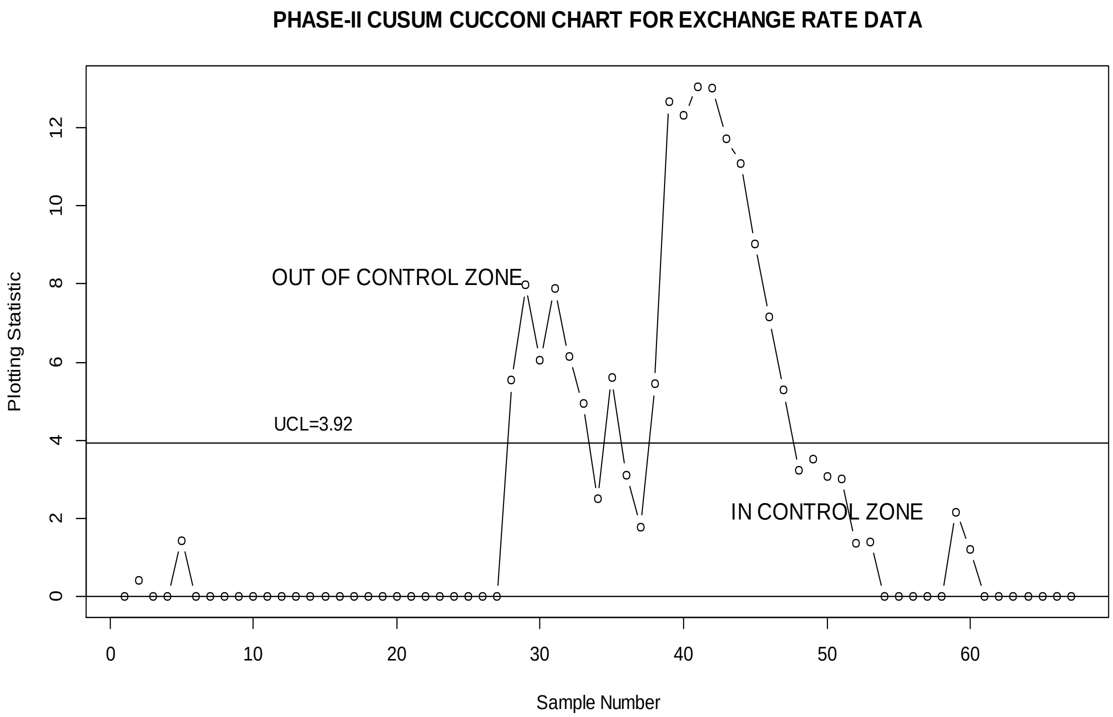
Figure 1. CUSUM CUCCONI CHARTS FOR EXCHANGE RATE DATA

2. A. CASE WITH $k = 0$



2. B. CASE WITH $k = 1.5$



2. C. CASE WITH $k = 3$

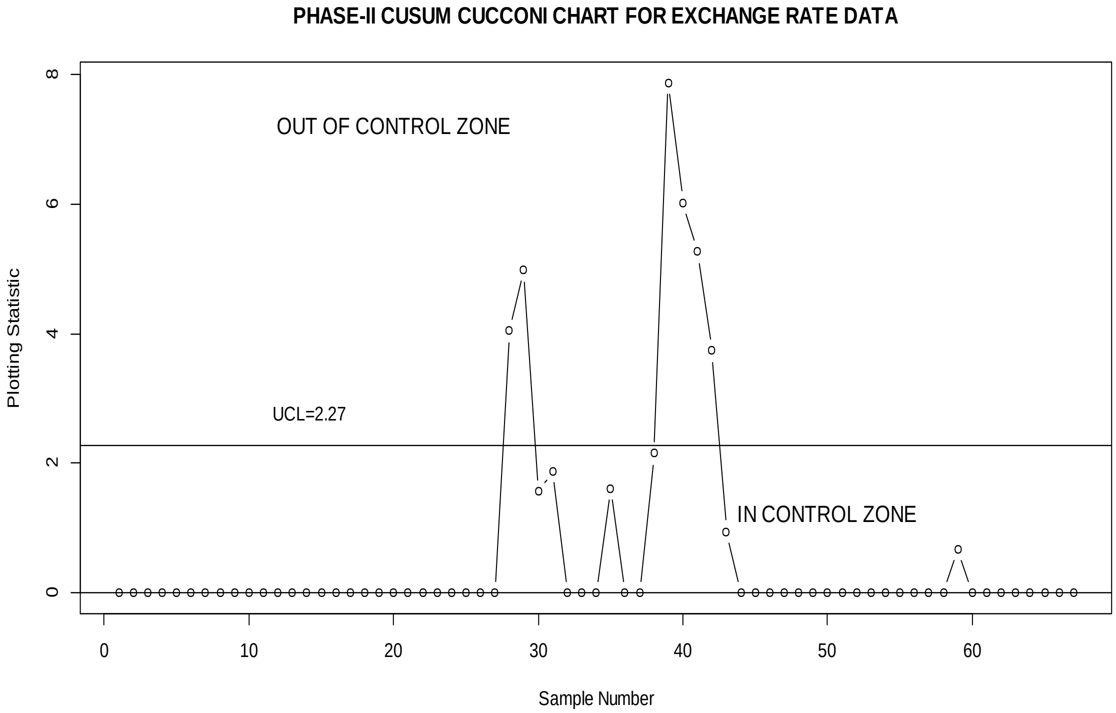


Table-1. Charting constant *H* for the CC chart, for values of *m* and *n*, and for some standard (target) values of ARL_0

Chart Parameter	Reference Sample Size	Test Sample size	The Charting Constant (Upper Control Limit) : *H*		
k	*M*	*n*	Target $ARL_0 = 250$	Target $ARL_0 = 370$	Target $ARL_0 = 500$
0	50	5	7.7284	8.8379	9.8327
		11	7.2668	8.1770	8.9850
	100	5	9.6293	11.1379	12.4718
		11	9.5207	10.9068	12.1032
	150	5	10.5284	12.2971	13.8690
		11	10.6476	12.4249	13.8856
	300	5	11.9452	14.2447	16.1410
		11	12.1344	14.3937	16.3412
1.5	50	5	2.2030	2.6025	2.9261
		11	1.9620	2.2300	2.4441
	100	5	2.8386	3.3011	3.6399
		11	2.4084	2.7586	3.0428
	150	5	3.0756	3.5461	3.9173
		11	2.6507	3.0484	3.3748
	300	5	3.3175	3.8253	4.2202
		11	2.9593	3.4145	3.7810
3	50	5	0.6457	1.0286	1.3408
		11	0.3852	0.6380	0.8399
	100	5	1.2437	1.6783	2.0200
		11	0.7944	1.1149	1.3878
	150	5	1.4601	1.9120	2.2703
		11	1.0268	1.4000	1.7101
	300	5	1.6994	2.1803	2.5484
		11	1.3460	1.7861	2.1331

Table-2. IC performance characteristics of the CC chart for ARL_0 = 500

M	n	H	ARL_0	$SDRL_0$	5th Percentile	1st Quartile	Median	3rd Quartile	95th Percentile
Simulated values with k=0									
50	5	9.8327	502.8624	972.8935	16	51	136	431	2560
50	11	8.9850	501.3713	972.8623	13	41	125	442	2559
100	5	12.4718	504.0649	879.7457	27	77	183	492	2219
100	11	12.1032	507.3778	868.4424	23	69	179	518	2223
150	5	13.8690	499.4431	817.8912	34	90	206	512	2056
150	11	13.8856	510.3034	799.5942	32	91	215	550	2044
300	5	16.1410	505.2865	716.6678	48	123	258	567	1820
300	11	16.3412	501.6056	677.7166	48	124	263	586	1770
Simulated values with k=1.5									
50	5	2.9261	499.2659	860.9454	8	60	184	516	2171
50	11	2.4441	499.5508	828.8751	6	54	183	552	2128
100	5	3.6400	503.3297	753.1925	12	86	239	584	1914
100	11	3.0428	501.8708	794.4779	9	70	211	565	2038
150	5	3.9173	497.9873	676.2686	15	101	267	614	1755
150	11	3.3748	494.9344	728.8364	12	84	236	587	1880
300	5	4.2202	495.1961	593.0698	19	119	301	645	1626
300	11	3.7810	496.2005	643.5127	16	106	278	627	1724
Simulated values with k=3									
50	5	1.3408	501.1229	854.1553	10	65	191	520	2144
50	11	0.8399	502.4958	826.2032	8	58	188	557	2140
100	5	2.0200	496.4873	717.9705	14	92	249	593	1826
100	11	1.3878	500.8659	784.5804	12	75	216	562	1990
150	5	2.2703	501.3947	659.0295	18	105	278	629	1736
150	11	1.7101	500.8260	740.0172	14	87	239	587	1912
300	5	2.5484	501.3179	590.1057	21	122	311	660	1628
300	11	2.1331	508.6202	651.5977	18	119	288	644	1745

Table-3. Performance comparisons for m=100, n=5 between various competetive charts for the Normal (θ,δ) distribution with ARL_0 = 500.

θ	Shewhart Lepage Chart	Shewhart Cucconi Chart	CUSUM Lepage chart			CUSUM Cucconi chart		
			k=0	k=3	k=6	k=0	k=1.5	k=3.0
				δ =0.50				
0	*>2200 (**)* *57, 419, 1478, 4959*	*>2200 (**)* *57, 444, 1598, #*	36.6 (154.0) 10, 15, 22, 33, 74	*>900 (**)* *10, 68 269 1080, #,*	*>2100 (**)* *48, 356, 1299, 4588, #*	36.0 (105.4) 14, 20, 26, 37, 71	*>4900 (**)* *#, #, #, #, #*	*>4900 (**)* *#, #, #, #, #*
0.25	*>2507.2(1960.7)* *83,604, 1994, , #, #*	*>2600 (**)* *83,675,2036, , #, #*	31.3 (74.7) 12, 18, 24, 34, 63	*>1400 (**)* *19, 154, 632, 2323, #*	*>2400 (**)* *73, 543, 1872, #, #*	30.0 (35.7) 15, 20, 25 34, 56	*>4700 (**)* *2721, #, #, #, #*	*>4700 (**)* *2588, #, #, #, #*
0.5	*>1600 (**)* *23, 201, 782, 2845,#*	*>1700 (**)* *23, 322,912,3895,#*	22.4 (9.1) 11, 17, 21, 26, 38	*>1200 (**)* *14, 117, 475, 1694, #*	*>1600 (**)* *22, 190, 761, 2790, #*	20.8 (7.6) 11, 16, 20, 24, 34	*>2800 (**)* *29, 441, 3048, #, #*	*>2900 (**)* *52, 595, 3508, , #, #*
0.75	264.3 (712.4) 2, 13, 46, 167, 1254	285.1(756.3) 2, 33, 65, 199, 1363	12.2 (5.2) 5, 8, 11, 15, 22	168.5 (570.97) 2, 7, 20, 77, 730	257.6 (705.7) 2, 12, 42, 158, 1215	11.6 (4.6) 5, 8, 11, 14, 20	399.2 (1042.9) 2, 9, 34, 178, 2777	*575.9 (1199.3)* *2, 21, 90, 402, 4325*
1.0	18.0 (76.6) 1, 2, 5, 14, 61	23.0 (82.3) 1, 2, 8, 32, 78	6.1 (2.6) 3, 4, 6, 7, 11	7.3 (33.1) 1, 2, 3, 6, 20	15.9 (65.9) 1, 2, 5, 12, 55	6.0 (2.6) 2, 4, 6, 7, 11	13.3 (113.8) 1, 2, 4, 8, 30	36.3 (192.7) 1, 2, 6, 19, 116
1.5	1.3 (0.8) 1, 1, 1, 1, 3	1.4(0.9) 1, 1, 1, 1, 3	2.6 (0.7) 2, 2, 2, 3, 4	1.2 (0.5) 1, 1, 1, 1,2	1.3 (0.7) 1, 1, 1, 1, 2	2.5 (0.8) 2, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.4 (0.9) 1, 1, 1, 2, 3
2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.1) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.7 (0.5) 1, 1, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
				δ =1.00				
0	503.2 (679.3) 18, 103, 274, 625, 1767	495.5 (709.9) 16, 93, 249, 591, 1831	498.98 (851.2) 35, 89, 198, 485, 2101	494.2 (670.97) 18, 101, 263, 610, 1750	510.6 (679.1) 19, 105, 280, 634, 1790	504.1 (879.7) 27, 77, 183, 492, 2219	503.3 (753.2) 12, 86, 239, 584, 1914	496.5 (718.0) 14, 92,249,593,1826
0.25	238.4 (379.6) 8, 43, 116, 277, 861	258.1 (417.7) 8, 45, 121, 295, 961	170.8 (384.4) 18, 40, 73, 151, 578	223.4 (361.9) 7, 39, 106, 257, 826	238.1 (372.3) 8, 44, 117, 279, 854	170.4 (388.7) 15, 35, 68, 148, 606	254.1 (444.9) 5, 38, 110, 280, 963	267.1 (448.2) 7, 43, 122, 303, 985
0.5	63.8 (98.0) 3, 13, 33, 76, 224	71.1 (120.9) 3, 13, 34, 81, 251	32.3 (38.8) 8, 16, 24, 38, 80	53.96 (89.9) 2, 11, 27, 62, 191	62.1 (98.9) 2, 12, 32, 73, 216	30.2 (42.8) 6, 14, 22, 35, 77	60.8 (114.4) 2, 10, 27, 67, 224	70.5 (125.7) 2, 12, 34, 81, 252
0.75	19.0 (25.4) 1, 5, 11, 24, 63	20.7 (29.8) 1, 5, 11, 25, 70	13.0 (7.8) 5, 8, 11, 16, 27	14.6 (19.4) 1, 4, 8, 18, 48	18.1 (24.9) 1, 4, 10, 22, 60	11.8 (7.5) 3, 7, 10, 15, 25	14.9 (22.2) 1, 3, 8,, 18, 50	19.5 (30.9) 1, 4, 10, 23, 66
1.0	7.3 (8.4) 1, 2, 5, 9, 22	7.7 (9.3) 1, 2, 5, 10, 24	7.2 (3.4) 3, 5, 7, 9, 14	5.4 (5.7) 1, 2, 4, 7, 16	6.9 (7.9) 1, 2, 4, 9, 21	6.6 (3.5) 2, 8, 6, 8, 13	5.3 (6.0) 1, 2, 3, 7, 16	7.0 (8.7) 1, 2, 4, 9, 22
1.5	2.1 (1.7) 1, 1, 2, 3, 5	2.1 (1.7) 1, 1, 1, 3, 5	3.5 (1.2) 2, 3, 3, 4, 6	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 2, 2, 5	3.2 (1.3) 2, 2, 3, 4, 6	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.4) 1, 1, 2, 2, 5
2	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.4 (0.6) 2, 2, 2, 3, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.1 (0.7) 1, 2, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.05) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.05) 1, 1 1, 1, 1	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
				δ =1.25				
0	105.1 (130.4) 5, 26, 63, 135, 346	75.4 (96.0) 4, 18, 44, 96, 248	55.4 (60.3) 11, 23, 38, 62, 141	90.6 (114.3) 4, 21, 54, 116, 300	101.4 (124.3) 5, 25, 61, 131, 329	39.3 (41.4) 8, 18, 29, 47, 103	63.0 (87.7) 2, 13, 35, 79, 215	73.6 (95.7) 3, 17, 43, 94, 248
0.25	67.1 (86.0) 3, 16, 40, 86, 220	54.3 (70.0) 3, 13, 32, 69, 180	38.3 (35.7) 10, 19, 29, 46, 94	56.96 (75.3) 3, 13, 33, 71, 191	65.7 (84.5) 3, 15, 39, 84, 220	29.4 (27.5) 7, 14, 23, 36, 74	44.5 (63.7) 2, 9, 24, 56, 151	53.2 (71.0) 2, 12, 31, 67, 181
0.5	29.0 (35.9) 2, 7, 17,37, 95	26.0 (32.7) 1, 6, 16, 33, 85	20.3 (13.9) 6, 11, 17, 25, 45	23.7 (30.4) 2, 6, 14, 30, 77	27.9 (34.5) 2, 7, 17, 36, 91	16. 5 (11.7) 4, 9, 14, 21, 37	20.5 (28.1) 2, 5, 12, 26, 68	24.9 (32.4) 2, 6, 15, 32, 81

0.75	12.9 (15.0) 1, 4, 8, 17, 40	12.2 (14.3) 1, 3, 8, 16, 39	11.4 (6.2) 4, 7, 10, 14, 23	10.2 (11.4) 1, 3, 7, 13, 31	12.4 (14.2) 1, 3, 8, 16, 38	9.7 (5.8) 2, 6, 9, 12, 20	9.0 (10.6) 1, 2, 6, 11, 28	11.2 (13.4) 1, 3, 7, 14, 36
1.0	6.4 (6.7) 1, 2, 4, 8, 19	6.2 (6.5) 1, 2, 4,8, 18	7.3 (3.5) 3, 5, 7, 9, 14	5.1 (4.9) 1, 2, 4, 6, 14	6.1 (6.3) 1, 2, 4, 8, 18	6.3 (3.4) 2, 4, 6, 8, 13	4.6 (4.5) 1, 2, 3, 6, 13	5.6 (6.0) 1, 2, 4, 7, 17
1.5	2.4 (2.0) 1, 1, 2, 3, 6	2.3 (1.9) 1, 1, 2, 3, 6	3.8 (1.5) 2, 3, 4, 5, 7	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6	3.4 (1.6) 2, 2, 3, 4, 6	2.0 (1.3) 1, 1, 2, 2, 5	2.2 (1.7) 1, 1, 2, 3, 5
2	1.4 (0.8) 1, 1, 1, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	2.6 (0.8) 2, 2, 2, 3, 4	1.4 (0.6) 1, 1, 1, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	2.3 (0.9) 1, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.7) 1, 1, 1, 2, 3
3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.1) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.4 (0.5) 1, 1, 1, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
δ =1.50								
0	38.3 (42.9) 2, 10, 24, 51, 121	24.4 (27.6) 2, 7, 15, 32, 77	21.7 (13.1) 8, 13, 19, 27, 46	29.4 (33.9) 2, 8, 18, 38, 93	36.9 (41.6) 2, 10, 24, 49, 116	15.2 (9.5) 4, 9, 13, 19, 33	17.8 (21.2) 2, 5, 11, 23, 57	22.8 (26.4) 2, 6, 14, 30, 73
0.25	29.4 (33.2) 2, 8, 19, 39, 93	20.4 (23.0) 1, 6, 13, 27, 64	18.9 (10.96) 7, 11, 17, 23, 39	22.8 (25.8) 2, 6, 15, 30, 72	28.2 (31.9) 2, 8, 18, 37, 88	13.7 (8.3) 4, 8., 12, 17, 29	15.1 (17.5) 1, 4, 9, 20, 48	19.3 (22.9) 1, 5, 12, 25, 62
0.5	17.2 (19.0) 1, 5, 11, 23, 54	13.6 (15.0) 1, 4, 9, 18, 42	13.9 (7.5) 5, 9, 12, 17, 28	13.5 (14.7) 1, 4, 9, 18, 42	16.7 (18.7) 1, 5, 11, 22, 52	10.7 (6.3) 3, 6, 10, 14, 22	9.8 (10.9) 1, 3, 6, 13, 30	12.6 (14.4) 1, 3, 8, 16, 40
0.75	9.7 (10.2) 1, 3, 6, 13, 29	8.3 (8.8) 1, 3, 5, 11, 25	9.7 (4.8) 4, 6, 9, 12, 19	7.8 (7.7) 1, 3, 5, 10, 22	9.3 (9.8) 1, 3, 6, 12, 28	7.8 (4.3) 2, 5, 7, 10, 16	6.2 (6.3) 1, 2, 4, 8, 18	7.6 (8.2) 1, 2, 5, 10, 23
1.0	5.8 (5.8) 1, 2, 4, 8, 17	5.2 (5.1) 1, 2, 4, 7, 15	6.99 (3.2) 3, 5, 6, 9, 13	4.7 (4.3) 1, 2, 3, 6, 13	5.6 (5.4) 1, 2, 4, 7, 16	5.8 (3.1) 2, 4, 5, 8, 11	3.9 (3.5) 1, 2, 3, 5, 11	4.7 (4.6) 1, 2, 3, 6, 14
1.5	2.6 (2.2) 1, 1, 2, 3, 7	2.4 (2.0) 1, 1, 2, 3, 6	4.1 (1.6) 2, 3, 4, 5, 7	2.3 (1.6) 1, 1, 2, 3, 5	2.5 (2.0) 1, 1, 2, 3, 6	3.5 (1.7) 2, 2, 3, 5, 7	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6
2	1.6 (1.0) 1, 1, 1, 2, 4	1.5 (0.9) 1, 1, 1, 2, 3	2.9 (0.9) 2, 2, 3, 3, 5	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (0.9) 1, 1, 1, 2, 3	2.5 (1.0) 1, 2, 2, 3, 4	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.8) 1, 1, 1, 2, 3
3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 1	2.1 (0.3) 2, 2, 2, 2, 3	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.5 (0.6) 1, 1, 1, 2, 2	1.0 (0.2) 1, 1, 1, 1, 2	1.0 (0.2) 1, 1, 1, 1, 1
δ =1.75								
0	19.4 (20.6) 1, 6, 13, 26, 60	11.7 (12.4) 1, 3, 8, 16, 36	13.5 (6.6) 5, 9, 12, 17, 26	13.8 (14.2) 1, 4, 9, 18, 41	18.3 (19.5) 1, 5, 12, 24, 57	9.3 (5.0) 3, 6, 9, 12, 18	8.2 (8.6) 1, 2, 5, 11, 25	10.7 (11.5) 1, 3, 7, 14, 33
0.25	16.5 (17.7) 1, 5, 11, 22, 51	10.7 (11.1) 1, 3, 7, 14, 32	12.5 (6.1) 5, 8, 11, 15, 24	11.8 (12.1) 1, 4, 8, 15, 35	15.6 (16.3) 1, 5, 10, 21, 47	8.9 (4.8) 2, 6, 8, 11, 18	7.5 (7.6) 1, 2, 5, 10, 22	9.9 (10.7) 1, 3, 6, 13, 30
0.5	11.6 (12.1) 1, 3, 8, 15, 35	8.4 (8.6) 1, 3, 6, 11, 25	10.4 (4.99) 4, 7, 9, 13, 20	8.8 (8.8) 1, 3, 6, 12, 26	11.1 (11.3) 1, 3, 7, 15, 33	7.7 (4.1) 2, 5, 7, 10, 15	6.0 (5.8) 1, 2, 4, 8, 17	7.5 (7.9) 1, 2, 5, 10, 23
0.75	7.9 (7.8) 1, 3, 5, 10, 23	6.1 (6.0) 1, 2, 4, 8, 18	8.3 (3.8) 3, 6, 8, 10, 15	6.1 (5.7) 1, 2, 4, 8, 17	7.5 (7.5) 1, 2, 5, 10, 22	6.4 (3.3) 2, 4, 6, 8, 13	4.5 (4.1) 1, 2, 3, 6, 12	5.5 (5.5) 1, 2, 4, 7, 16
1.0	5.2 (5.0) 1, 2, 4, 7, 15	4.4 (4.2) 1, 2, 3, 6, 13	6.5 (2.9) 3, 4, 6, 8, 12	4.3 (3.6) 1, 2, 3, 6, 11	5.0 (4.6) 1, 2, 4, 7, 14	5.2 (2.7) 2, 3, 5, 7, 10	3.4 (2.9) 1, 2, 2, 4, 9	4.0 (3.7) 1, 2, 3, 5, 11
1.5	2.7 (2.3) 1, 1, 2, 4, 7	2.5 (2.0) 1, 1, 2, 3, 6	4.3 (1.7) 2, 3, 4, 5, 7	2.4 (1.7) 1, 1, 2, 3, 6	2.6 (2.1) 1, 1, 2, 3, 7	3.6 (1.7) 2, 2, 3, 5, 7	2.1 (1,4) 1, 1, 2, 3, 5	2.3 (1.8) 1, 1, 2, 3, 6
2	1.7 (1.1) 1, 1, 1, 2, 4	1.6 (1.0) 1, 1, 1, 2, 4	3.1 (1.1) 2, 2, 3, 4, 5	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.1) 1, 1, 1, 2, 4	2.6 (1.1) 1, 2, 2, 3, 5	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (0.9) 1, 1, 1, 2, 3
3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.2 (0.4) 2, 2, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.7 (0.7) 1, 1, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
δ =2.00								

0	11.8 (12.0) 1, 4, 8, 16, 36	7.1 (7.0) 1, 2, 5, 9, 21	9.8 (4.3) 4, 7, 9, 12, 18	8.2 (7.8) 1, 3, 6, 11, 23	11.1 (11.2) 1, 3, 8, 15, 33	6.9 (3.5) 2, 4, 6, 9, 13	5.1 (4.6) 1, 2, 4, 7, 14	6.4 (6.4) 1, 2, 4, 8, 19
0.25	10.6 (10.6) 1, 3, 7, 14, 32	6.8 (6.8) 1, 2, 5, 9, 20	9.4 (4.1) 4, 6, 9, 12, 17	7.7 (7.2) 1, 3, 5, 10, 22	10.0 (10.1) 1, 3, 7, 13, 30	6.6 (3.4) 2, 4, 6, 9, 13	4.8 (4.3) 1, 2, 3, 6, 13	6.0 (6.1) 1, 2, 4, 8, 18
0.5	8.5 (8.4) 1, 3, 6, 11, 25	5.8 (5.6) 1, 2, 4, 8, 17	8.4 (3.7) 4, 6, 8, 10, 15	6.3 (5.8) 1, 2, 5, 8, 18	8.1 (8.0) 1, 3, 6, 11, 24	6.1 (3.1) 2, 4, 6, 9, 12	4.3 (3.7) 1, 2, 3, 6, 11	5.3 (5.2) 1, 2, 4, 7, 15
0.75	6.3 (6.1) 1, 2, 4, 8, 18	4.8 (4.4) 1, 2, 3, 6, 13	7.2 (3.1) 3, 5, 7, 9, 13	4.9 (4.2) 1, 2, 4, 6, 13	6.1 (5.7) 1, 2, 4, 8, 17	5.4 (2.7) 2, 3, 5, 7, 10	3.6 (3.0) 1, 2, 3, 5, 10	4.3 (4.0) 1, 2, 3, 6, 12
1.0	4.7 (4.4) 1, 2, 3, 6, 13	3.8 (3.4) 1, 1, 3, 5, 10	6.1 (2.6) 3, 4, 6, 8, 11	3.8 (3.1) 1, 2, 3, 5, 10	4.5 (4.2) 1, 2, 3, 6, 13	4.7 (2.3) 2, 3, 4, 6, 9	3.0 (2.3) 1, 1, 2, 4, 8	3.4 (3.0) 1, 1, 2, 4, 9
1.5	2.8 (2.3) 1, 1, 2, 4, 7	2.4 (1.9) 1, 1, 2, 3, 6	4.3 (1.7) 2, 3, 4, 5, 7	2.4 (1.7) 1, 1, 2, 3, 6	2.7 (2.1) 1, 1, 2, 3, 7	3.5 (1.7) 2, 2, 3, 5, 7	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.7) 1, 1, 2, 3, 6
2	1.9 (1.3) 1, 1, 1, 2, 4	1.7 (1.1) 1, 1, 2, 3, 4	3.3 (1.1) 2, 2, 3, 4, 5	1.7 (1.0) 1, 1, 1, 2, 4	1.8 (1.2) 1, 1, 1, 2, 4	2.7 (1.2) 1, 2, 2, 3, 5	1.6 (0.9) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 1, 2
3	1.2 (0.5) 1,1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2	2.3 (0.5) 2, 2, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	1.8 (0.7) 1, 1, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2

** indicates variance estimate is not meaningful and # indicates the percentile value exceeds 5000.

Table-4. Performance comparisons for m=300, n=5 between various competetive charts for the Normal (θ,δ) distribution with ARL_0 = 500.

θ	Shewhart Lepage Chart	Shewhart Cucconi Chart	CUSUM Lepage chart			CUSUM Cucconi chart		
			k=0	k=3	k=6	k=0	k=1.5	k=3.0
				δ =0.50				
0	>2600 (**) 139, 860, 2414, #, #	>2700 (**) 139, 920, 2645, #, #	26.9 (12.5) 13, 19, 24, 32, 50	>1000 (**) 22, 134, 415, 1221, #	>3700 (**) 328, 2210, #, #, #	35.1 (12.8) 21, 27, 33, 41, 58	>5000 (**) #, #, #, #, #	>5000 (**) #, #, #, #, #
0.25	>3200 (**) 226, 1408, 3947, #, #	>3200 (**) 226, 1464, 4002, , #, #	25.7 (6.8) 16, 21, 25, 30, 38	>2100 (**) 62, 417, 1382, 4161, #	>4100 (**) 600, 3892, #, #, #	34.6 (10.4) 22, 27, 33, 40, 53	>4900 (**) #, #, #, #, #	>4900 (**) #, #, #, #, #
0.5	>1800 (**) 54, 354, 1087, 2987, #	>1800 (**) 54, 374, 1155, 3100, #	13.3 (4.2) 7, 10, 13, 16, 21	>1900 (**) 53, 362, 1181, 3344, #	>2400 (**) 84, 613, 1999, #, #	26.1 (6.3) 17, 22, 26, 30, 37	>3000 (**) 102, 877, 3407, #, #	>3100 (**) 147, 1097, 3798, #, #
0.75	114.0 (235.5) 3, 17, 47, 118, 423	136.3 (235.5) 3, 19, 52, 129, 452	6.5 (2.0) 4, 5, 6, 8, 10	70.5 (197.2) 3,. 9, 22, 59, 266	166.3 (364.4) 4, 22, 61, 162, 634	13.6 (4.2) 7, 11, 13, 16, 21	120.8 (355.3) 2, 11, 30, 90, 486	266.2 (560.9) 4, 29, 89, 249, 1078
1.0	9.4 (13.3) 1, 2, 5, 11, 31	10.4 (18.0) 1, 2, 5, 11, 31	2.8 (0.6) 2, 2, 3, 3, 4	4.5 (4.0) 1, 2, 3, 5, 11	9.5 (14.5) 1, 3, 5, 11, 31	6.7 (2.3) 3, 5, 7, 8, 11	5.1 (5.8) 1, 2, 4, 6, 13	12.1 (22.6) 1, 2, 6, 13, 42
1.5	1.2 (0.5) 1, 1, 1, 1, 2	1.3 (0.6) 1, 1, 1, 1, 3	2.0 (0.1) 2, 2, 2, 2, 2	1.3 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.7 (0.8) 2, 2, 3, 3, 4	1.3 (0.5) 1, 1, 1, 2, 2	1.3 (0.6) 1, 1, 1, 1, 2
2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.3) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.00 1, 1, 1, 1, 1
				δ =1.00				
0	499.6 (556.6) 24, 129, 319, 669, 1583	499.4 (586.5) 22, 125, 308, 652, 1620	494.2 (646.2) 58, 140, 277, 575, 1660	499.6 (568.6) 23, 125, 315, 663, 1601	498.8 (562.2) 23, 129, 317, 663, 1590	505.3 (716.7) 48, 123,258,567,1820	495.2 (593.1) 19,119,301,645,1626	501.3 (590.1) 21,122,311,660.1628

0.25	208.8 (246.1) 9, 51, 129, 273,687	222.98 (276.7) 10, 53, 133, 288, 740	131.3 (154.7) 28, 55, 90, 152, 364	199.0 (243.0) 9, 48, 119, 258, 656	208.6 (252.0) 10, 51, 127, 271, 683	126.1 (163.0) 23, 49, 82, 144, 359	210.0 (272.8) 7, 46, 121, 268, 711	221.9 (275.5) 8, 52, 132, 289, 739
0.5	53.4 (61.4) 3, 14, 34, 70, 172	58.5 (69.4) 3, 15, 36, 76, 188	32.3 (18.4) 12 20 28 40 66	45.0 (53.6) 3, 12, 28, 59, 144	52.2 (60.3) 3, 14, 33, 69, 167	29.5 (17.9) 9, 18, 26, 37, 62	46.6 (59.5) 2, 11, 28, 60, 154	55.8 (67.1) 2, 13, 34, 73, 182
0.75	17.1 (18.5) 1, 5, 11, 23, 53	18.3 (20.1) 1, 5, 12, 24, 57	14.3 (6.2) 6, 10, 13, 18, 26	12.6 (13.3) 1, 4, 8, 16, 37	16.1 (17.3) 1, 5, 11, 21, 50	12.9 (6.3) 4, 9, 12, 16, 24	12.2 (13.2) 1, 3, 8, 16, 37	16.2 (18.3) 1, 4, 10, 22, 51
1.0	6.8 (6.7) 1, 2, 5, 9, 20	7.1 (7.2) 1, 2, 5, 9, 21	8.1 (3.1) 4, 6, 8, 10, 14	4.9 (4.2) 1, 2, 4, 6, 13	6.2 (6.0) 1, 2, 4, 8, 18	7.3 (3.3) 2, 5, 7, 9, 13	4.7 (4.1) 1, 2, 3, 6, 13	6.0 (6.1) 1, 2, 4, 8, 18
1.5	2.0 (1.5) 1, 1, 2, 3, 5	2.05 (1.5) 1, 1, 2, 3, 5	3.9 (1.2) 2, 3, 4, 5, 6	1.8 (1.0) 1, 1, 2, 2, 4	1.9 (1.3) 1, 1, 2, 2, 4	3.5 (1.4) 2, 2, 3, 4, 6	1.8 (1.0) 1, 1, 2, 2, 4	1.9 (1.2) 1, 1, 2, 2, 4
2	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.6 (0.6) 2, 2, 3, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.4 (0.7) 1, 2, 2, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.04) 1, 1, 1, 1, 1	2.0 (0.09) 2, 2, 2, 2, 2	1.0 (0.05) 1, 1, 1, 1, 1	1.0 (0.05) 1, 1, 1, 1, 1	1.5 (0.5) 1, 1, 1, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =1.25								
0	101.8 (108.1) 5, 28, 68, 137, 312	77.2 (83.4) 4, 21, 50, 104, 240	55.8 (35.6) 18, 32, 47, 69, 122	87.9 (94.99) 5, 24, 58, 118, 274	99.8 (106.2) 5, 27, 66, 135, 310	40.2 (25.8) 12, 23, 34, 50, 87	60.5 (67.7) 2, 13, 39, 81, 193	73.1 (80.5) 3, 19, 47, 98, 230
0.25	64.6 (69.6) 3, 18, 43, 87, 201	53.4 (58.5) 3, 15, 35, 71, 166	40.4 (23.3) 14, 25, 35, 50, 85	53.8 (59.2) 3, 15, 35, 71, 170	62.7 (67.5) 4, 17, 41, 84, 195	30.6 (18.1) 9, 18, 27, 39, 64	41.2 (46.7) 2, 10, 26, 55, 132	50.1 (55.8) 2, 13, 32, 67, 157
0.5	27.4 (29.3) 2, 8, 18, 37, 85	24.7 (26.3) 2, 7, 16, 33, 76	22.2 (10.9) 9, 14, 20, 28, 43	22.0 (23.1) 2, 7, 15, 29, 67	26.4 (28.0) 2, 7, 17, 35, 81	18.0 (9.6) 6, 11, 16, 23, 36	18.0 (19.7) 2, 5, 12, 24, 56	22.8 (25.1) 2, 6, 15, 31, 72
0.75	12.4 (12.6) 1, 4, 8, 17, 37	11.5 (11.6) 1, 4, 8, 16, 34	12.8 (5.6) 5, 9, 12, 16, 23	9.4 (9.0) 1, 3, 7, 13, 27	11.7 (11.8) 1, 4, 8, 16, 35	10.8 (5.3) 3, 7, 10, 14, 21	8.1 (8.0) 1, 2, 6, 11, 24	10.2 (10.6) 1, 3, 7, 14, 31
1.0	6.2 (5.9) 1, 2, 4, 8, 18	5.98 (5.7) 1, 2, 4, 8, 17	8.2 (3.3) 4, 6, 8, 10, 14	4.9 (4.1) 1, 2, 4, 6, 13	5.9 (5.5) 1, 2, 4, 8, 17	7.0 (3.3) 2, 5, 7, 9, 13	4.3 (3.6) 1, 2, 3, 6, 11	5.2 (4.9) 1, 2, 4, 7, 15
1.5	2.4 (1.8) 1, 1, 2, 3, 6	2.3 (1.8) 1, 1, 2, 3, 6	4.4 (1.5) 2, 3, 4, 5, 7	2.1 (1.3) 1, 1, 2, 3, 5	2.3 (1.7) 1, 1, 2, 3, 6	3.8 (1.6) 2, 2, 4, 5, 7	2.0 (1.2) 1, 1, 2, 2, 4	2.1 (1.5) 1, 1, 2, 3, 5
2	1.4 (0.7) 1, 1, 1, 2, 3	1.3 (0.7) 1, 1, 1, 2, 3	2.9 (0.8) 2, 2, 3, 3, 4	1.4 (0.6) 1, 1, 1, 2, 3	1.4 (0.7) 1, 1, 1, 2, 3	2.6 (0.9) 1, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.6) 1, 1, 1, 2, 2
3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.1 (0.2) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.6 (0.5) 1, 1, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
δ =1.50								
0	38.3 (39.3) 2, 11, 26, 52, 117	25.2 (25.7) 2, 7, 17, 34, 36	24.0 (11.2) 10, 16, 22, 30, 45	28.3 (28.7) 2, 9, 19, 38, 84	36.1 (37.1) 2, 10, 25, 49, 109	17.1 (8.5) 6, 11, 16, 22, 33	17.1 (17.7) 2, 5, 11, 23, 52	22.6 (24.1) 2, 6, 15, 31, 69
0.25	29.1 (30.3) 2, 8, 19, 39, 88	20.9 (21.5) 1, 6, 14, 28, 63	21.2 (9.8) 9, 14, 19, 26, 40	22.0 (22.2) 2, 7, 15, 30, 66	27.8 (28.7) 2, 8, 19, 38, 84	15.3 (7.6) 5, 10, 14, 19, 29	14.3 (14.7) 2, 4, 10, 19, 43	18.7 (19.7) 2, 5, 12, 25, 57
0.5	17.1 (17.3) 1, 5, 12, 23, 51	13.5 (13.7) 1, 4, 9, 18, 41	15.7 (6.98) 7, 11, 15, 19, 29	12.9 (12.6) 1, 4, 9, 17, 38	16.1 (16.3) 1, 5, 11, 22, 48	11.9 (5.8) 4, 8, 11, 15, 23	9.3 (9.2) 1, 3, 6, 13, 27	12.1 (12.4) 1, 3, 8, 16, 36
0.75	9.6 (9.4) 1, 3, 7, 13, 28	8.3 (8.1) 1, 3, 6, 11, 24	11.1 (4.7) 5, 8, 10, 14, 20	7.4 (6.7) 1, 3, 5, 10, 21	9.1 (8.8) 1, 3, 6, 12, 26	8.8 (4.2) 3, 6, 8, 11, 16	5.8 (5.2) 1, 2, 4, 8, 16	7.2 (7.3) 1, 2, 5, 10, 21
1.0	5.7 (5.3) 1, 2, 4, 8, 16	5.1 (4.7) 1, 2, 4, 7, 14	8.0 (3.2) 4, 6, 8, 10, 14	4.6 (3.7) 1, 2, 4, 6, 12	5.5 (4.97) 1, 2, 4, 7, 15	6.5 (3.1) 2, 4, 6, 8, 12	3.8 (3.2) 1, 2, 3, 5, 10	4.5 (4.1) 1, 2, 3, 6, 13
1.5	2.6 (2.1) 1, 1, 2, 3, 7	2.4 (1.9) 1, 1, 2, 3, 6	4.7 (1.7) 2, 3, 4, 6, 8	2.3 (1.5) 1, 1, 2, 3, 5	2.5 (1.9) 1, 1, 2, 3, 6	4.0 (1.7) 2, 3, 4, 5, 7	2.1 (1.3) 1, 1, 2, 2, 5	2.2 (1.6) 1, 1, 2, 3, 6
2	1.6 (1.0) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3	3.2 (1.0) 2, 3, 3, 4, 5,	1.5 (0.8) 1, 1, 1, 2, 3	1.5 (0.9) 1, 1, 1, 2, 3	2.8 (1.1) 1, 2, 3, 3, 5	1.4 (0.7) 1, 1, 1, 2, 3	1.5 (0.8) 1, 1, 1, 2, 3
3	1.1 (0.3) 1, 1, 1, 1, 2	1.0 (0.2) 1, 1, 1, 1, 1	2.2 (0.4) 2, 2, 2, 2, 3	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2	1.8 (0.6) 1, 1, 2, 2, 3	1.0 (0.2) 1, 1, 1, 1, 2	1.0 (0.2) 1, 1, 1, 1, 1
δ =1.75								
0	19.3 (19.3) 1, 6, 13, 26, 58	12.2 (12.0) 1, 4, 8, 16, 36	15.3 (6.3) 7, 11, 14, 19, 27	13.2 (12.5) 2, 5, 9, 18, 38	18.1 (18.1) 1, 6, 12, 24, 54	10.6 (5.0) 4, 7, 10, 13, 20	7.9 (7.5) 1, 2, 6, 11, 23	10.6 (10.7) 1, 3, 7, 14, 32

0.25	16.7 (16.7) 1, 5, 11, 23, 50	11.0 (10.9) 1, 3, 8, 15, 33	14.3 (5.9) 6, 10, 13, 18, 25	11.7 (11.2) 1, 4, 8, 16, 33	15.4 (15.4) 1, 5, 11, 21, 46	10.1 (4.7) 3, 7, 10, 13, 19	7.3 (6.8) 1, 2, 5, 10, 21	9.5 (9.5) 1, 3, 6, 13, 29
0.5	11.7 (11.5) 1, 4, 8, 16, 35	8.5 (8.3) 1, 3, 6, 12, 25	11.98 (4.9) 5, 8, 11, 15, 21	8.6 (7.8) 1, 3, 6, 11, 24	10.98 (10.7) 1, 4, 8, 15, 32	8.8 (4.1) 3, 6, 8, 11, 16	5.8 (5.2) 1, 2, 4, 8, 16	7.4 (7.3) 1, 2, 5, 10, 22
0.75	7.8 (7.5) 1, 3, 6, 11, 23	6.1 (5.8) 1, 2, 4, 8, 18	9.5 (3.9) 4, 7, 9, 12, 17	5.98 (5.1) 1, 2, 4, 8, 16	7.3 (6.96) 1, 2, 5, 10, 21	7.2 (3.4) 2, 5, 7, 9, 13	4.4 (3.7) 1, 2, 3 , 6, 12	5.4 (5.1) 1, 2, 4, 7, 15
1.0	5.3 (4.8) 1, 2, 4, 7, 15	4.4 (4.0) 1, 2, 3, 6, 12	7.5 (2.96) 3, 5, 7, 9, 13	4.2 (3.4) 1, 2, 3, 6, 11	5.0 (4.5) 1, 2, 4, 7, 14	5.9 (2.7) 2, 4, 6, 8, 11	3.4 (2.6) 1, 2, 2, 4, 9	3.9 (3.4) 1, 2, 3, 5, 11
1.5	2.7 (2.2) 1, 1, 2, 4, 7	2.5 (1.9) 1, 1, 2, 3, 6	4.9 (1.8) 2, 4, 5, 6, 8	2.4 (1.6) 1, 1, 2, 3, 5	2.6 (2.0) 1, 1, 2, 3, 7	4.0 (1.8) 2, 2, 4, 5, 7	2.1 (1.4) 1, 1, 2, 3, 5	2.3 (1.6) 1, 1, 2, 3, 6
2	1.7 (1.1) 1, 1, 1, 2, 4	1.6 (1.0) 1, 1, 1, 2, 4	3.5 (1.2) 2, 3, 3, 4, 6	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.0) 1, 1, 1, 2, 4	2.9 (1.2) 1, 1, 2, 2, 3	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (0.9) 1, 1, 1, 2, 3
3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.3 (0.5) 2, 2, 2, 3, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	1.9 (0.7) 1, 1, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
δ =2.00								
0	12.0 (11.7) 1, 4, 8, 16, 35	7.4 (7.0) 1, 2, 5, 10, 21	11.3 (4.3) 5, 8, 11, 14, 19	8.1 (7.2) 1, 3, 6, 11, 22	10.9 (10.5) 1, 3, 8, 15, 32	7.8 (3.5) 3, 5, 7, 10, 14	5.0 (4.2) 1, 2, 4, 7, 13	6.3 (6.0) 1, 2, 4, 8, 18
0.25	10.9 (10.6) 1, 3, 8, 15, 32	6.98 (6.6) 1, 2, 5, 9, 20	10.9 (4.2) 5, 8, 10, 13, 19	7.5 (6.5) 1, 3, 6, 10, 20	9.99 (9.6) 1, 3, 7, 14, 29	7.6 (3.4) 2, 5, 7, 10, 14	4.8 (4.0) 1, 2, 4, 6, 13	6.0 (5.7) 1, 2, 4, 8, 17
0.5	8.7 (8.3) 1, 3, 6, 12, 25	5.96 (5.6) 1, 2, 4, 8, 17	9.7 (3.8) 5, 7, 9, 12, 17	6.2 (5.3) 1, 3, 5, 8, 17	8.1 (7.6) 1, 3, 6, 11, 23	7.0 (3.2) 2, 5, 7, 9, 13	4.2 (3.4) 1, 2, 3, 6, 11	5. 2 (4.8) 1, 2, 4, 7, 15
0.75	6.5 (6.1) 1, 2, 5, 9, 19	4.8 (4.3) 1, 2, 3, 6, 13	8.3 (3.2) 4, 6, 8, 10, 14	4.9 (4.0) 1, 2, 4, 6, 13	6.0 (5.6) 1, 2, 4, 8, 17	6.2 (2.8) 2, 4, 6, 8, 11	3.5 (2.8) 1, 2, 3, 5, 9	4.2 (3.7) 1, 2, 3, 6, 12
1.0	4.8 (4.3) 1, 2, 3, 6, 13	3.8 (3.3) 1, 1, 3, 5, 10	7.0 (2.7) 3, 5, 7, 9, 12	3.8 (2.9) 1, 2, 3, 5, 10	4.5 (3.98) 1, 2, 3, 6, 13	5.4 (2.4) 2, 4, 5, 7, 10	3.0 (2.2) 1, 1, 2, 4, 7	3.4 (2.8) 1, 1, 2, 4, 9
1.5	2.8 (2.3) 1, 1, 2, 4, 7	2.4 (1.9) 1, 1, 2, 3, 6	4.96 (1.8) 2, 4, 5, 6, 8	2.4 (1.6) 1, 1, 2, 3, 6	2.7 (2.1) 1, 1, 2, 3, 7	4.0 (1.8) 2, 2, 4, 5, 7	2.1 (1.3) 1, 1, 2, 2, 5	2.2 (1.6) 1, 1, 2, 3, 5
2	1.9 (1.3) 1, 1, 1, 2, 4	1.7 (1.1) 1, 1, 1, 2, 4	3.7 (1.2) 2, 3, 4, 4, 6	1.7 (1.0) 1, 1, 1, 2, 4	1.8 (1.2) 1, 1, 1, 2, 4	3.0 (1.3) 2, 2, 3, 4, 5	1.6 (0.9) 1, 1, 1, 2, 3	1.6 (1.0) 1, 1, 1, 2, 4
3	1.2 (0.5) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	2.5 (0.6) 2, 2, 2, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.0 (0.7) 1, 2, 2, 2, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2

** indicates variance estimate is not meaningful and # indicates the percentile value exceeds 5000.

Table-5. Performance comparisons for m=100, n=5 between various competetive charts for the Laplace (θ, δ) distribution with ARL_0 = 500.

θ	Shewhart Lepage Chart	Shewhart Cucconi Chart	CUSUM Lepage chart			CUSUM Cucconi chart		
			k=0	k=3	k=6	k=0	k=1.5	k=3.0
δ =0.50								
0	*>2300 (**)* *77, 533, 1793, #, #*	*>2500 (**)* *77, 545, 1821, #, #*	92.0 (365.0) 13, 23, 35, 61, 206	*>1400 (**)* *22, 160, 589, 2111, #*	*>2300 (**)* *69, 501, 1669, #, #*	124.2 (419.6) 20, 32, 49, 82, 304	*>4800 (**)* *4385, #, #, #, #*	*>4900 (**)* *#, #, #, #, #*
0.25	*>3000 (**)* *133, 1000, 3343, #, #*	*>3100 (**)* *133, 1074, 3347, #, #*	86.1 (273.8) 17, 29, 44, 71, 202	*>2300 (**)* *51, 461, 1748, #, #*	*>2900 (**)* *125, 953, 3213, #, #*	61.3 (131.1) 20, 30, 42, 62, 138	*>4400 (**)* *813, #, #, #, #*	*>4800 (**)* *#, #, #, #, #*
0.5	*>2100 (**)* *33, 313, 1385, #, #*	*>2300 (**)* *33, 333, 1451, #, #*	39.2 (31.8) 14, 24, 33, 47, 82	*>1900 (**)* *20, 211, 1039, 4352, #*	*>2200 (**)* *31, 310, 1382, #, #*	29.8 (15.2) 13, 21, 27, 35, 55	*>3000 (**)* *41, 625, 3966, #, #*	*>4300 (**)* *489, #, #, #, #*
0.75	*>800 (**)* *4, 35, 153, 716, #*	*>800 (**)* *4, 55, 187, 818, #*	18.3 (10.3) 6, 11, 16, 23, 38	*>600 (**)* *3, 14, 63, 385, #*	*>700 (**)* *4, 31, 143, 695, #*	15.9 (7.1) 6, 11, 15, 20, 29	*>1100 (**)* *3, 26 168, 1338, #*	*>2900 (**)* *26, 501, 3786, #, #*
1.0	173.9 (597.4) 1, 5, 19, 78, 749	177.8 (617.0) 1, 6, 22, 81, 752	9.1 (4.8) 4, 6, 8, 11, 18	86.8 (446.6) 1, 3, 7, 19, 254	167.0 (599.0) 1, 5, 16, 67, 707	9.0 (4.2) 4, 6, 8, 11, 17	195.5 (752.9) 2, 4, 10, 39, 869	*>1200 (**)* *2, 25, 194, 1801, #*

1.5	5.8 (50.6) 1, 1, 2, 4, 15	5.9 (50.9) 1, 1, 3, 4, 17	3.9 (1.4) 2, 3, 4, 4, 6	2.1 (3.7) 1, 1, 2, 2, 5	4.6 (42.5) 1, 1, 2, 3, 10	3.9 (1.6) 2, 3, 4, 5, 7	3.1 (39.6) 1, 1, 2, 3, 6	51.5 (373.6) 1, 2, 3, 7, 79
2	1.3 (1.7) 1, 1, 1, 1, 2	1.4 (1.8) 1, 1, 1, 1, 2	2.5 (0.6) 2, 2, 2, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.7) 1, 1, 1, 1, 2	2.5 (0.7) 2, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 1, 2	1.9 (9.1) 1, 1, 1, 2, 4
3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.1) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.6 (0.5) 1, 1, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
δ =1.00								
0	509.6(676.4) 18, 107, 280, 631, 1794	496.7 (712.7) 16, 94,250, 593,1811	502.2 (862.8) 34, 89, 195, 483, 2118	495.2 (676.3) 17, 100, 264, 606, 1777	506.7 (677.8) 18, 106, 275, 630, 1772	507.2 (889.5) 27, 76, 181, 487, 2254	501.9 (740.4) 13, 87, 243, 594, 1886	502.9 (719.8) 15, 93, 253, 601, 1857
0.25	347.2(528.8) 10, 62, 167, 411, 1271	377.9 (602.8) 10,63,173,429,1429	266.7 (572.7) 21, 48, 97, 230, 1008	333.3 (535.9) 9, 54, 151, 381, 1249	347.2 (531.8) 10, 61, 167, 408, 1282	249.0 (542.9) 17, 42, 87, 213, 969	374.4 (618.6) 7,55, 161, 416, 1470	376.9 (598.9) 10, 61, 170, 432, 1438
0.5	146.0 (273.3) 4, 23, 63, 157, 548	179.0 (344.7) 4, 25, 73, 189, 674	60.2 (138.9) 10, 20, 33, 59, 171	128.99 (268.2) 4, 18, 49, 130, 495	144.9 (278.7) 4, 22, 60, 152, 545	54.2 (121.9) 8, 18, 31, 54, 157	161.4 (339.0) 2, 19, 58, 160, 641	182.4 (361.8) 3, 24, 72, 188, 708
0.75	51.5 (109.5) 2, 8, 22, 54, 187	70.3 (158.9) 2, 10, 28, 70, 265	19.5 (18.4) 6, 10, 15, 23, 46	39.0 (96.7) 2, 6, 15, 37, 142	49.3 (103.8) 2, 8, 21, 51, 179	18.4 (17.5) 5, 9, 14, 22, 44	52.1 (140.3) 2, 6, 17, 47, 196	67 (155.1) 2, 9, 25, 67, 254
1.0	18.7(36.5) 1, 4, 9, 20, 66	25.8 (57.7) 1, 4, 11, 26, 94	10.1 (5.99) 4, 6, 9, 12, 21	12.1 (24.1) 1, 3, 6, 13, 41	17.5 (35.4) 1, 3, 8, 19, 62	9.8 (6.1) 3, 6, 8, 12, 21	15.4 (42.0) 1, 3, 6, 14, 52	23.5 (51.2) 1, 3, 9, 24, 86
1.5	3.9 (5.2) 1, 1, 2, 5, 12	4.8 (7.4) 1, 1 ,3,5, 16	4.7 (1.9) 2, 3, 4, 6, 8	2.8 (2.5) 1, 1, 2, 3, 7	3.5 (4.3) 1, 1, 2, 4, 10	4.5 (2.1) 2, 3, 4, 6, 8	3.0 (3.2) 1, 1, 2, 4, 8	4.1 (6.6) 1, 1, 2, 5, 13
2	1.7 (1.3) 1, 1, 1, 2, 4	1.8 (1.6) 1, 1, 1,2, 5	3.1 (0.98) 2, 2, 3, 4, 5	1.6 (0.8) 1, 1, 1, 2, 3	1.7 (1.1) 1, 1, 1, 2, 4	2.9 (1.1) 2, 2, 3, 4, 5	1.6 (0.9) 1, 1, 1, 2, 3	1.7 (1.3) 1, 1, 1, 2, 4
3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.1 (0.3) 2, 2, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	1.8 (0.6) 1, 1, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
δ =1.25								
0	155.4 (203.5) 7, 36, 90, 198, 515	121.1 (167.4) 5,27,68,150,414	92.1 (152.1) 16, 33, 55, 99, 271	137.4 (181.0) 6, 31, 79, 174, 460	153.6 (197.2) 6, 35, 90, 195, 518	69.0 (118.8) 11, 24, 41, 74, 206	109.0 (161.6) 3, 21, 58, 132, 386	119.7 (166.3) 4, 25, 66, 150, 407
0.25	116.5 (163.9) 5, 25, 64, 144, 398	99.9 (141.4) 4, 21, 54, 122, 348	64.8 (102.1) 12, 25, 40, 70, 184	102.4 (149.1) 4, 21, 54, 124, 361	116.2 (160.6) 4, 25, 64, 145, 402	50.4 (77.6) 9, 19, 32, 56, 144	87.4 (136.5) 2, 16, 44, 104, 310	100.7 (149.5) 3, 20, 54, 123, 347
0.5	62.5 (95.6) 3, 13, 33, 74, 218	60.8 (97.1) 2, 12, 31, 72, 213	30.6 (34.6) 8, 15, 22, 36, 76	50.2 (78.9) 2, 10, 25, 59, 177	60.0 (93.6) 2, 12, 31, 71, 214	26. 2 (31.1) 6, 12, 19, 31, 67	48.8 (84.1) 2, 8, 23, 55, 177	59.6 (97.3) 2, 11, 30, 69, 213
0.75	28.4 (43.2) 1, 6, 15, 34, 98	31.4 (51.1) 1, 6, 16, 36, 111	15.9 (12.7) 5, 9, 13, 19, 36	20.7 (35.2) 2, 5, 11, 24, 72	26.9 (40.9) 1, 6, 14, 32, 92	14.2 (10.4) 4, 8, 12, 18, 32	21.7 (38.5) 2, 4, 10, 24, 77	29.4 (49.7) 1, 5, 14, 34, 106
1.0	13.2 (19.2) 1, 3, 7, 16, 44	15.0 (23.0) 1, 3, 8, 17, 52	9.6 (5.3) 4, 6, 8, 12, 19	9.2 (13.4) 1, 3, 5, 11, 29	12.5 (19.0) 1, 3, 7, 15, 41	8.9 (5.4) 2, 5, 8, 11, 19	9.5 (15.3) 1, 2, 5, 11, 32	13.8 (21.6) 1, 3, 7, 16, 48
1.5	3.9 (4.3) 1, 1, 3, 5, 12	4.3 (5.3) 1, 1, 3, 5, 13	4.9 (2.0) 2, 3, 5, 6, 9	2.9 (2.5) 1, 1, 2, 4, 7	3.7 (4.1) 1, 1, 2, 5, 11	4.8 (2.2) 2, 3, 4, 6, 9	3.0 (2.7) 1, 1, 2, 4, 8	3.8 (4.6) 1, 1, 2, 4, 11
2	1.9 (1.5) 1, 1, 1, 2, 5	2.0 (1.7) 1, 1, 1, 2, 5	3.3 (1.1) 2, 2, 3, 4, 5	1.7 (1.0) 1, 1, 1, 2, 4	1.9 (1.4) 1, 1, 1, 2, 4	3.0 (1.2) 2, 2, 3, 4, 5	1.7 (1.0) 1, 1, 1, 2, 4	1.8 (1.4) 1, 1, 1, 2, 4
3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	2.2 (0.5) 2, 2, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	1.9 (0.7) 1, 2, 2, 2, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2
δ =1.50								
0	68.0 (81.8) 3, 17, 42, 89, 221	47.4 (59.7) 2, 12, 29, 61, 156	34.1 (28.5) 10, 18, 27, 41, 80	54.8 (68.4) 3, 13, 32, 70, 181	65.8 (79.4) 3, 16, 40, 85, 214	24.8 (21.0) 6, 13, 20, 31, 60	37.3 (50.2) 2, 8, 21, 47,127	45.8 (58.1) 2, 10, 27, 58, 153
0.25	55.4 (69.2) 3, 13, 33, 71, 182	41.8 (52.9) 2, 10, 25, 54, 137	28.8 (23.4) 9, 16, 23, 35, 67	43.7 (57.1) 3, 11, 26, 55, 145	53.1 (66.9) 3, 13, 31, 68, 176	21.8 (17.5) 5, 11, 18, 27, 51	32.2 (44.5) 2, 7, 18, 40, 109	40.0 (51.9) 2, 9, 23, 51, 134
0.5	34.5 (45.1) 2, 8, 20, 43, 115	29.6 (38.5) 2,7, 17, 37, 100	19.6 (13.8) 6, 11, 16, 24, 43	26.4 (35.7) 2, 6, 15, 32, 89	33.6 (44.0) 2, 8, 19, 42, 112	15.9 (11.6) 4, 9, 13, 20, 36	21.1 (30.1) 2, 5, 12, 26, 72	27.8 (38.6) 2, 6, 16, 34, 94
0.75	19.4 (25.8) 1, 5, 11, 24, 63	17.9 (24.3) 1, 4, 10, 22, 59	12.8 (7.8) 5, 8, 11, 16, 27	13.8 (17.9) 1, 4, 8, 17, 44	18.2 (24.2) 1, 4, 11, 23, 61	10.9 (6.8) 3, 6, 10, 14, 23	12.3 (17.3) 1, 3, 7, 15, 41	16. 6 (23.5) 1, 4, 9, 20, 56

1.0	10.6 (13.3) 1, 3, 6, 13, 34	10.6 (14.2) 1, 3, 6, 13, 34	8.8 (4.5) 4, 6, 8, 11, 17	7.4 (8.7) 1, 2, 5, 9, 22	9.9 (12.5) 1, 3, 6, 12, 32	7.9 (4.4) 2, 5, 7, 10, 16	7.0 (8.7) 1, 2, 4, 8, 22	9.4 (12.6) 1, 2, 5, 12, 31
1.5	4.0 (4.2) 1, 1, 3, 5, 12	4.0 (4.4) 1, 1, 3, 5, 12	5.0 (2.1) 2, 4, 5, 6, 9	3.0 (2.5) 1, 1, 2, 4, 8	3.7 (3.6) 1, 1, 3, 5, 10	4.5 (2.2) 2, 3, 4, 6, 9	2.9 (2.5) 1, 1, 2, 4, 7	3.5 (3.8) 1, 1, 2, 4, 10
2	2.1 (1.7) 1, 1, 1, 3, 5	2.1 (1.7) 1, 1, 1,2, 5	3.5 (1.2) 2, 3, 3, 4, 6	1.8 (1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 2, 2, 5	3.2 (1.3) 2, 2, 3, 4, 6	1.8 (1.1) 1, 1, 2, 2, 4	1.9 (1.5) 1, 1, 1, 2, 5
3	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.3 (0.6) 2, 2, 2, 3, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.0 (0.7) 1, 2, 2, 2, 3	1.2 (0.4) 1, 1, 1,1, 2	1.2 (0.4) 1, 1, 1, 1, 2
				δ =1.75				
0	36.6 (41.6) 2, 10, 23, 48, 116	24.1 (27.8) 2, 6, 15, 32, 76	20.2 (12.2) 7, 12, 17, 25, 42	27.4 (31.8) 2, 8, 17, 35, 87	35.2 (40.3) 2, 9, 22, 47, 111	14.7 (9.2) 4, 9, 13, 19, 32	17.3 (21.2) 2, 4, 11, 22, 56	22.6 (26.9) 2, 5, 14, 29, 74
0.25	31.9 (36.7) 2, 8, 20, 42, 103	21.9 (25.8) 1, 6, 14, 28, 69	18.2 (10.7) 6, 11, 16, 23, 38	23.7 (27.98) 2, 7, 15, 30, 75	30.2 (35.4) 2, 8, 19, 39, 97	13.6 (8.5) 4, 8, 12, 17, 29	15.7 (19.7) 2, 4, 9, 20, 51	20.5 (24.8) 1, 5, 12, 27, 67
0.5	22.5 (26.6) 1, 6, 14, 29, 72	17.0 (20.2) 1, 5, 10, 22, 54	14.3 (8.0) 5, 9, 13, 18, 29	16.1 (19.1) 2, 5, 10, 20, 51	21.4 (25.5) 1, 6, 13, 28, 68	11.2 (6.6) 3, 7, 10, 14, 23	11.9 (14.4) 1, 3, 7, 15, 38	20.5 (24.8) 1, 5, 12, 27, 67
0.75	14.3 (16.8) 1, 4, 9, 18, 46	11.9 (14.0) 1, 3, 7, 15, 38	10.7 (5.5) 4, 7, 10, 13, 21	10.1 (11.6) 1, 3, 6, 13, 31	13.4 (15.8) 1, 4, 8, 17, 43	8.8 (5.0) 2, 5, 8, 11, 18	8.0 (9.5) 1, 2, 5, 10, 25	10.8 (13.3) 1, 3, 6, 14, 35
1.0	8.9 (10.2) 1, 3, 6, 11, 28	8.0(9.3) 1, 2, 5, 10, 25	8.1 (3.8) 3, 5, 7, 10, 15	6.2 (6.5) 1, 2, 4, 8, 18	8.3 (9.4) 1, 2, 5, 11, 26	6.8 (3.7) 2, 4, 6, 9, 14	5.4 (5.9) 1, 2, 4, 7, 16	7.2 (8.6) 1, 2, 4, 9, 23
1.5	3.9 (3.9) 1, 1, 3, 5, 11	3.7 (3.8) 1, 1, 2, 5, 11	5.0 (2.0) 2, 4, 5, 6, 9	3.0 (2.4) 1, 1, 2, 4, 8	3.7 (3.5) 1, 1, 3, 5, 10	4.4 (2.1) 2, 3, 4, 6, 8	2.8(2.2) 1, 1, 2, 3, 7	3.4 (3.3) 1, 1, 2, 4, 9
2	2.2 (1.8) 1, 1, 2, 3, 6	2.1 (1.7) 1, 1, 2, 3, 5	3.6 (1.3) 2, 3, 3, 4, 6	1.8 (1.1) 1, 1, 2, 2, 4	2.1 (1.6) 1, 1, 2, 3, 5	3.2 (1.4) 2, 2, 3, 4, 6	1.8(1.1) 1, 1, 2, 2, 4	2.0 (1.5) 1, 1, 2, 2, 5
3	1.3 (0.6) 1, 1, 1, 1, 2	1.2 (0.6) 1, 1, 1, 1, 2	2.5 (0.7) 2, 2, 2, 3, 4	1.3 (0.5) 1, 1, 1, 1, 2	1.3 (0.6) 1, 1, 1, 1, 2	2.1 (0.8) 1, 2, 2, 2, 4	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
				δ =2.00				
0	23.2 (25.3) 2, 6, 15, 31, 72	14.6 (16.0) 1, 4, 10, 19, 45	14.5 (7.5) 6, 9, 13, 18, 28	16.3 (17.6) 2, 5, 11, 21, 50	21.9 (24.2) 2, 6, 14, 29, 68	10.4 (5.8) 3, 6, 9, 13, 21	10.1 (11.1) 1, 3, 7, 13, 31	13.3 (15.1) 1, 3, 8, 17, 42
0.25	20.4 (22.6) 1, 6, 13, 27, 64	13.7(15.2) 1, 4, 9, 18, 42	13.5 (6.9) 5, 9, 12, 17, 26	14.4 (15.6) 1, 4, 9, 19, 44	19.4 (21.5) 1, 5, 13, 26, 61	10.0 (5.6) 3, 6, 9, 13, 20	9.3 (10.3) 1, 3, 6, 12, 29	12.5 (14.2) 1, 3, 8, 16, 40
0.5	15.9 (17.7) 1, 4, 10, 21, 49	11.4(12.9) 1, 3, 7, 15, 35	11.5 (5.7) 5, 7, 10, 14, 22	11.1 (11.9) 1, 4, 7, 14, 33	14.9 (16.6) 1, 4, 10, 20, 46	8.7 (4.8) 2, 5, 8, 11, 18	7.7 (8.5) 1, 2, 5, 10, 23	10.3 (11.8) 1, 3, 6, 13, 33
0.75	11.1 (12.3) 1, 3, 7, 15, 34	8.7 (10.0) 1, 3, 6, 11, 27	9.2 (4.4) 4, 6, 8, 11, 18	7.8 (8.2) 1, 3, 5, 10, 23	10.4 (11.5) 1, 3, 7, 14, 32	7.3 (3.9) 2, 5, 7, 9, 14	5.9(6.0) 1, 2, 4, 7, 17	7.8 (8.8) 1, 2, 5, 10, 24
1.0	7.6 (8.3) 1, 2, 5, 10, 23	6.3(7.0) 1, 2, 4, 8, 19	7.4 (3.3) 3, 5, 7, 9, 13	5.4 (5.2) 1, 2, 4, 7, 15	7.1 (7.7) 1, 2, 5, 9, 21	6.1 (3.1) 2, 4, 6, 8, 12	4.4 (4.3) 1, 2, 3, 6, 12	5.6 (6.2) 1, 2, 4, 7, 17
1.5	3.8(3. 7) 1, 1, 3, 5, 11	3.4(3.3) 1, 1, 2, 4, 10	4.99 (1.99) 2, 4, 5, 6, 9	2.98 (2.3) 1, 1, 2, 4, 7	3.5 (3.3) 1, 1, 3, 5, 10	4.3 (2.0) 2, 3, 4, 5, 8	2.6 (2.0) 1, 1, 2, 3, 6	3.1(2.9) 1, 1, 2, 4, 9
2	2.3 (1.8) 1, 1, 2, 3, 6	2.1 (1.7) 1, 1, 2, 3, 5	3.7 (1.3) 2, 3, 3, 4, 6	1.99 (1.2) 1, 1, 2, 2, 4	2.2 (1.7) 1, 1, 2, 3, 5	3.2 (1.4) 2, 2, 3, 4, 6	1.9(1.1) 1, 1, 2. 2, 4	2.0 (1.4) 1, 1, 2, 2, 5
3	1.3 (0.7) 1, 1, 1, 2, 3	1.3 (0.6) 1, 1, 1, 1, 3	2.6 (0.7) 2, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.6) 1, 1, 1, 2, 3	2.2 (0.8) 1, 2, 2, 3, 4	1.3(0.5) 1, 1, 1, 1, 2	1.3 (0.6) 1, 1, 1, 1, 2

** indicates variance estimate is not meaningful and # indicates the percentile value exceeds 5000.

Table-6. Performance comparisons for *m*=300, *n*=5 between various competetive charts for the Laplace (θ, δ) distribution with ARL_0 = 500.

θ			CUSUM Lepage chart			CUSUM Cucconi chart		

	Shewhart Lepage Chart	Shewhart Cucconi Chart	*k*=0	*k*=3	*k*=6	*k*=0	*k*=1.5	*k*=3.0
				δ =0.50				
0	*>2900 (**)* *177, 1063, 2954, #, #*	*>3000 (**)* *177, 1152, 3085, #, #*	47.1 (39.9) 18, 28, 39, 55, 100	*>1800 (**)* *60, 376, 1149, 3175, #*	*>3900 (**)* *457, 3000, #, #, #*	76.1 (74.5) 31, 46, 62, 87, 158	*>4900 (**)* *#, #, #, #, #*	*>4900 (**)* *#, #, #, #, #*
0.25	*>3800 (**)* *434, 2630, #, #, #*	*>3800 (**)* *434, 2638, #, #, #*	61.3 (40.9) 24, 38, 52, 73, 127	*>3700 (**)* *300, 2146, #, #, #*	*>4400 (**)* *1004, #, #, #, #*	62.5 (31.5) 31, 43, 56, 73, 115	*>4700 (**)* *2578, #, #, #, #*	*>4700 (**)* *2195, #, #, #, #*
0.5	*>2200 (**)* *77, 533, 1686, 4807, #*	*>2300 (**)* *77, 542, 1685, 4822, #*	44.0 (17.7) 21, 32, 41, 53, 76	*>2700 (**)* *92, 721, 2587, 5000, #*	*>2900 (**)* *126, 888, 2989, #, #*	37.5 (12.2) 21, 29, 36, 44, 60	*>3200 (**)* *147, 1226, 4556, #, #*	*>3300 (**)* *195, 1407, 4715, #, #*
0.75	455.6 (823.7) 8, 51, 157, 461, 2002	459.6 (832.3) 8, 50, 162, 468, 2035	19.3 (7.8) 9, 14, 18, 23, 34	384.9 (861.0) 5, 24, 80, 290, 1963	*693.4 (1122.7)* *11, 72, 237, 733, 3517*	19.0 (6.5) 10, 14, 18, 23, 31	*662.0 (1210.9)* *5, 39, 151, 589, 4246*	*>1000 (**)* *15, 123, 409, 1281, #*
1.0	54.6 (134.6) 2, 7, 20, 52, 206	62.6 (138.0) 2, 8, 21, 54, 210	9.5 (3.3) 5, 7, 9, 11, 16	16.0 (58.0) 2, 4, 7, 14, 47	71.9 (213.9) 2, 7, 20, 57, 277	10.1 (3.5) 5, 8, 10, 12, 17	29.5 (133.5) 2, 4, 9, 20, 91	133.6 (371.5) 2, 10, 34, 105, 537
1.5	2.6 (3.0) 1, 1, 2, 3, 7	2.7 (3.0) 1, 1, 3, 3, 7	4.1 (1.0) 3, 3, 4, 5, 6	2.0 (1.0) 1, 1, 2, 2, 4	2.4 (2.2) 1, 1, 2, 3, 6	4.2 (1.3) 2, 3, 4, 5, 7	2.2 (1.2) 1, 1, 2, 3, 5	2.9 (3.4) 1, 1, 2, 3, 8
2	1.2 (0.4) 1, 1, 1, 1, 2	1.3 (0.4) 1, 1, 1, 1, 2	2.7 (0.6) 2, 2, 3, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2	2.6 (0.7) 2, 2, 3, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.1) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.20 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
				δ =1.00				
0	502.8 (563.0) 23, 129, 321, 673, 1607	500.3 (587.1) 22, 123, 310, 656, 1638	501.7 (649.6) 59, 141, 283, 586, 1689	505.6 (577.4) 24, 129, 318, 669, 1629	498.6 (560.1) 23, 130, 318, 666, 1575	503.7 (718.1) 49,124,258,562,1802	499.6 (601.97) 19, 119, 299, 652, 1646	505.5 (593.0) 21, 124, 312, 665, 1639
0.25	305.5 (370.8) 13, 74, 184, 398, 997	341.2 (424.7) 14, 79, 201, 440, 1133	202.6 (279.5) 34, 70, 122, 225, 622	303.0 (380.7) 13, 70, 178, 388,1019	310.8 (382.9) 13, 74, 187, 403,1018	185.1 (718.1) 28, 61,107,203,578	325.8 (426.2) 10, 71,185, 414, 1112	336.8 (424.9) 13, 77, 197, 436,1119
0.5	111.8 (142.5) 5, 26, 65, 142, 372	139.2 (183.4) 6, 31, 80, 175, 470	47.99 (36.4) 15, 26, 39, 58, 110	96.1 (128.1) 5, 22, 54, 121, 325	109.7 (140.97) 5, 25, 64, 139, 371	44.6 (35.1) 13, 24,36,55,102	112.2 (161.8) 3, 22, 60, 138, 390	134.6 (182.7) 4, 29, 76, 168, 462
0.75	37.5 (46.99) 2, 9, 22, 48, 123	50.6 (67.1) 2, 12, 29, 63, 169	19.7 (9.7) 8, 13, 18, 24, 38	26.2 (34.0) 2, 7, 15, 32, 86	36.1 (46.3) 2, 9, 21, 46, 120	18.8 (9.8) 7, 12,17,23,37	31.6 (45.6) 2, 7, 17, 39, 108	45.3 (61.4) 2, 10, 25, 57, 154
1.0	13.97 (16.4) 1, 4, 9, 18, 44	18.8 (24.0) 1, 5, 11, 24, 61	10.9 (4.4) 5, 8, 10, 13, 19	8.7 (9.3) 1, 3, 6, 11, 25	12.9 (15.4) 1, 4, 8, 17, 41	10.5 (4.7) 4,7,10,13,19	9.8 (11.8) 1, 3, 6, 12, 30	15.7 (20.9) 1, 4, 9, 20, 52
1.5	3.3 (3.0) 1, 1, 2, 4, 9	3.98 (3.97) 1, 1, 3, 5, 11	5.2 (1.7) 3, 4, 5, 6, 8	2.5 (1.7) 1, 1, 2, 3, 6	3.1 (2.6) 1, 1, 2, 4, 8	4.9 (1.97) 2,4,5,6,8	2.7 (1.8) 1, 1, 2, 3, 6	3.2 (2.98) 1, 1, 2, 4, 9
2	1.7 (1.1) 1, 1, 1, 2, 4	1.7 (1.2) 1, 1, 1, 2, 4	3.5 (0.9) 2, 3, 3, 4, 5	1.5 (0.8) 1, 1, 1, 2, 3	1.6 (0.9) 1, 1, 1, 2, 3	3.2 (1.1) 2,2,3,4,5	1.6 (0.8) 1, 1, 1, 2, 3	1.6 (0.9) 1, 1, 1, 2, 3
3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2	2.3 (0.5) 2, 2, 2, 3, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.2 (0.5) 2, 2, 2, 2,3	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.2) 1, 1, 1, 1, 2
				δ =1.25				
0	150.1 (164.1) 7, 40, 98, 202, 468	122.99 (134.99) 6, 33, 80, 166, 385	84.5 (68.8) 24, 43, 66, 104, 203	135.8 (150.4) 7, 36, 88, 181, 427	147.5 (159.5) 7, 39, 97, 200, 457	62.1 (49.5) 16, 32, 49, 77, 149	103.1 (119.6) 3, 26, 65, 137, 330	119.8 (134.3) 5, 30, 77, 160, 383
0.25	109.1 (122.4) 6, 28, 70, 146, 342	97.2 (110.7) 5, 25, 62, 129, 308	59.3 (42.2) 18, 33, 49, 73, 137	94.7 (108.3) 5, 25, 60, 125, 304	108.0 (122.4) 5, 28, 69, 144, 341	46.8 (33.6) 13, 26, 38, 58, 107	78.6 (93.9) 3, 19, 48, 103, 258	93.98 (109.4) 4, 23, 59, 124, 301
0.5	53.9 (62.1) 3, 14, 34, 71, 174	54.5 (64.3) 3, 14, 34, 72, 174	30.4 (17.1) 11, 19, 27, 38, 62	42.6 (49.8) 3, 11, 26, 56, 136	52.3 (60.8) 3, 14, 33, 68, 169	26. 1 (15.3) 8, 16, 23, 33, 55	39.0 (48.1) 2, 9, 23, 51, 127	51.0 (61.98) 2, 12, 31, 67, 165
0.75	24.3 (27.5) 2, 7, 15, 32, 76	26.5 (31.4) 2, 7, 17, 34, 84	16.7 (7.8) 7, 11, 15, 21, 31	16.9 (18.9) 2, 5, 11, 22, 52	22.8 (25.98) 2, 6, 15, 30, 73	15.1 (7.6) 5, 10, 14, 19, 29	16.4 (19.3) 2, 4, 10, 21, 52	23.4 (27.9) 2, 6, 14, 31, 76
1.0	11.4 (12.3) 1, 3, 7, 15, 35	12.7 (14.1) 1, 4, 8, 17, 40	10.5 (4.3) 5, 7, 10, 13, 18	7.5 (7.4) 1, 3, 5, 10, 21	10.4 (11.2) 1, 3, 7, 14, 32	9.6 (4.4) 3, 7, 9, 12, 18	7.6 (7.9) 1, 2, 5, 10, 22	10.8 (12.5) 1, 3, 7, 14, 34
1.5	3.6 (3.3) 1, 1, 3, 5, 10	3.9 (3.7) 1, 1, 3, 5, 11	5.5 (1.9) 3, 4, 5, 7, 9	2.8 (1.9) 1, 1, 2, 4, 6	3.3 (2.8) 1, 1, 2, 4, 9	5.1 (2.1) 2, 4, 5, 6, 9	2.8 (1.97) 1, 1, 2, 4, 7	3.3 (2.9) 1, 1, 2, 4, 9
2	1.9 (1.3)	1.9 (1.4)	3.7 (1.1)	1.7 (0.9)	1.8 (1.1)	3.4 (1.3)	1.7 (0.9)	1.7 (1.1)

	1, 1, 1, 2, 4	1, 1, 1, 2, 5	2, 3, 4, 4, 6	1, 1, 1, 2, 3	1, 1, 1, 2, 4	2, 2, 3, 4, 6	1, 1, 1, 2, 3	1, 1, 1, 2, 4
3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.4 (0.6) 2, 2, 2, 3, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	2.3 (0.6) 2, 2, 2, 3, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
δ =1.50								
0	66.3 (70.2) 4, 18, 44, 90, 205	48.2 (51.7) 3, 13, 32, 65, 149	36.2 (19.6) 14, 23, 32, 45, 73	52.6 (56.3) 3, 15, 35, 71, 162	64.1 (67.9) 4, 18, 43, 87, 196	26.4 (14.7) 9, 16, 23, 33, 54	34.9 (39.1) 2, 9, 23, 47, 110	44.8 (48.9) 2, 12, 29, 61, 139
0.25	53.3 (56.9) 3, 15, 35, 72, 164	41.3 (44.4) 2, 11, 27, 56, 129	30.5 (15.9) 12, 20, 27, 38, 60	41.5 (45.3) 3, 12, 27, 55, 129	51.6 (55.0) 3, 14, 34, 70, 160	23.3 (12.8) 8, 15, 21, 29, 47	29.2 (32.6) 2, 8, 19, 39, 92	38.2 (42.1) 2, 10, 25, 52, 120
0.5	32.2 (35.2) 2, 9, 21, 43, 101	27.8 (29.9) 2, 8, 18, 37, 86	21.2 (10.1) 9, 14, 19, 26, 40	23.6 (25.4) 2, 7, 16, 31, 72	30.8 (33.5) 2, 8, 20, 41, 96	17.2 (8.8) 6, 11, 16, 22, 33	18.6 (20.7) 2, 5, 12, 25, 58	25.2 (28.1) 2, 6, 16, 34, 79
0.75	17.5 (18.7) 1, 5, 12, 23, 54	16.7 (18.1) 1, 5, 11, 22, 52	14.2 (6.2) 6, 10, 13, 17, 26	12.2 (12.5) 1, 4, 8, 16, 36	16.5 (17.7) 1, 5, 11, 22, 51	12.0 (5.8) 4, 8, 11, 15, 23	10.5 (11.1) 1, 3, 7, 14, 32	14.4 (16.0) 1, 4, 9, 19, 45
1.0	9.7 (10.1) 1, 3, 7, 13, 29	9.5 (10.0) 1, 3, 6, 13, 29	9.9 (3.9) 5, 7, 9, 12, 17	6.7 (6.1) 1, 3, 5, 9, 19	8.96 (9.2) 1, 3, 6, 12, 26	8.6 (3.9) 3, 6, 8, 11, 16	6.0 (5.7) 1, 2, 4, 8, 17	8.1 (8.7) 1, 2, 5, 11, 25
1.5	3.7 (3.4) 1, 1, 3, 5, 10	3.7 (3.4) 1, 1, 3, 5, 10	5.7 (1.8) 3, 4, 5, 7, 9	2.9 (2.1) 1, 1, 2, 4, 7	3.4 (2.9) 1, 1, 3, 4, 9	5.0 (2.1) 2, 3, 5, 6, 9	2.7 (1.95) 1, 1, 2, 4, 7	3.2 (2.7) 1, 1, 2, 4, 9
2	2.0 (1.5) 1, 1, 2, 3, 5	2.0 (1.4) 1, 1, 1, 2, 5	3.9 (1.2) 2, 3, 4, 5, 6	1.8 (1.0) 1, 1, 2, 2, 4	1.9 (1.3) 1, 1, 2, 2, 5	3.5 (1.3) 2, 2, 3, 4, 6	1.8 (0.99) 1, 1, 2, 2, 4	1.8 (1.2) 1, 1, 1, 2, 4
3	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2	2.6 (0.7) 2, 2, 3, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.3 (0.7) 2, 2, 2, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2
δ =1.75								
0	36.2 (37.1) 2, 10, 24, 49, 109	24.6 (25.4) 2, 7, 17, 33, 74	22.5 (10.3) 9, 15, 21, 28, 42	26.3 (27.0) 2, 8, 18, 35, 79	34.4 (35.6) 2, 10, 23, 47, 105	16.3 (8.1) 6, 11, 15, 21, 31	16.2 (17.0) 2, 5, 11, 22, 49	22.0 (23.3) 2, 6, 15, 30, 68
0.25	31.5 (32.8) 2, 9, 21, 42, 96	22.2 (23.0) 2, 6, 15, 30, 67	20.3 (9.2) 9, 14, 19, 25, 38	22.2 (22.6) 2, 7, 15, 30, 66	29.6 (30.7) 2, 9, 20, 40, 90	15.1 (7.5) 5, 10, 14, 19, 29	14.5 (15.3) 2, 4, 10, 19, 45	19.7 (21.2) 2, 5, 13, 27, 62
0.5	21.8 (22.5) 2, 6, 15, 30, 66	16.98 (17.7) 1, 5, 11, 23, 51	16.0 (6.96) 7, 11, 15, 20, 29	14.9 (15.1) 2, 5, 10, 20, 44	20.5 (21.5) 2, 6, 14, 28, 62	12.5 (6.0) 4, 8, 12, 16, 23	10.8 (11.2) 1, 3, 7, 14, 32	14.9 (15.96) 1, 4, 10, 20, 46
0.75	13.6 (14.0) 1, 4, 9, 18, 41	11.5 (11.8) 1, 3, 8, 15, 35	12.0 (4.9) 5, 9, 11, 15, 21	9.2 (8.8) 1, 3, 7, 12, 26	12.6 (12.8) 1, 4, 9, 17, 38	9.8 (4.5) 3, 7, 9, 12, 18	7.2 (7.0) 1, 2, 5, 10, 21	9.9 (10.3) 1, 3, 7, 13, 30
1.0	8.4 (8.3) 1, 3, 6, 11, 25	7.6 (7.6) 1, 2, 5, 10, 22	9.1 (3.5) 4, 7, 9, 11, 16	5.9 (5.1) 1, 2, 4, 8, 16	7.8 (7.7) 1, 3, 5, 10, 23	7.6 (3.4) 3, 5, 7, 10, 14	4.9 (4.3) 1, 2, 4, 6, 13	6.5 (6.5) 1, 2, 4, 9, 19
1.5	3.8 (3.4) 1, 1, 3, 5, 10	3.5 (3.2) 1, 1, 3, 5, 10	5.7 (2.0) 3, 4, 5, 7, 9	2.9 (2.0) 1, 1, 2, 4, 7	3.5 (2.9) 1, 1, 3, 4, 9	4.9 (2.1) 2, 3, 5, 6, 9	2.7 (1.9) 1, 1, 2, 3, 6	3.0 (2.5) 1, 1, 2, 4, 8
2	2.2 (1.6) 1, 1, 2, 3, 5	2.0 (1.5) 1, 1, 2, 3, 5	4.1 (1.3) 2, 3, 4, 5, 6	1.9 (1.1) 1, 1, 2, 2, 4	2.1 (1.4) 1, 1, 2, 3, 5	3.6 (1.4) 2, 2, 3, 4, 6	1.8 (1.0) 1, 1, 2, 2, 4	1.9 (1.2) 1, 1, 2, 2, 4
3	1.3 (0.6) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.7 (0.7) 2, 2, 3, 3, 4	1.3 (0.5) 1, 1, 1, 1, 2	1.3 (0.5) 1, 1, 1, 1, 2	2.4 (0.8) 2, 2, 2, 3, 4	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
δ =2.00								
0	23.2 (23.5) 2, 7, 16, 32, 69	15.0 (15.3) 1, 5, 10, 20, 45	16.3 (6.8) 7, 11, 15, 20, 29	15.5 (15.2) 2, 5, 11, 21, 45	21.5 (21.9) 2, 6, 15, 29, 64	11.8 (5.5) 4, 8, 11, 15, 22	9.5 (9.4) 1, 3, 7, 13, 28	13.0 (13.7) 1, 4, 9, 18, 40
0.25	20.6 (21.1) 2, 6, 14, 28, 61	13.9 (14.2) 1, 4, 9, 19, 41	15.3 (6.3) 7, 11, 14, 19, 27	13.9(13.6) 2, 5, 10, 19, 40	19.1 (19.4) 2, 6, 13, 26, 57	11.2 (5.2) 4, 8, 11, 14, 21	8.9 (8.7) 1, 3, 6, 12, 26	12.1 (12.6) 1, 3, 8, 16, 37
0.5	15.7 (15.9) 1, 5, 11, 21, 47	11.4 (11.5) 1, 4, 8, 15, 34	12.9 (5.3) 6, 9, 12, 16, 23	10.6 (10.1) 1, 4, 7, 14, 30	14.5 (14.6) 1, 4, 10, 20, 43	9.9 (4.6) 3, 7, 9, 12, 18	7.2 (6.8) 1, 2, 5, 10, 20	9.8 (10.1) 1, 3, 7, 13, 30
0.75	11.0 (11.1) 1, 3, 8, 15, 33	8.5 (8.5) 1, 3, 6, 11, 25	10.5 (4.1) 5, 8, 10, 13, 18	7.4 (6.6) 1, 3, 5, 10, 20	10.0 (9.9) 1, 3, 7, 13, 30	8.3 (3.7) 3, 6, 8, 10, 15	5.5 (4.9) 1, 2, 4, 7, 15	7.3 (7.3) 1, 2, 5, 10, 22
1.0	7.5 (7.3) 1, 2, 5, 10, 22	6.2 (5.95) 1, 2, 4, 8, 18	8.4 (3.2) 4, 6, 8, 10, 14	5.2 (4.3) 1, 2, 4, 7, 14	6.8 (6.4) 1, 2, 5, 9, 19	6.8 (3.0) 2, 5, 6, 9, 12	4.1 (3.4) 1, 2, 3, 5, 11	5.2 (4.98) 1, 2, 4, 7, 15
1.5	3.7 (3.3) 1, 1, 3, 5, 10	3.3 (2.9) 1, 1, 2, 4, 9	5.7 (1.98) 3, 4, 5, 7, 9	2.9 (2.0) 1, 1, 2, 4, 7	3.4 (2.9) 1, 1, 3, 4, 9	4.8 (2.0) 2, 3, 5, 6, 8	2.5 (1.7) 1, 1, 2, 3, 6	2.9 (2.4) 1, 1, 2, 4, 8

2	2.3 (1.7) 1, 1, 2, 3, 6	2.1 (1.6) 1, 1, 2, 3, 5	4.2 (1.4) 2, 3, 4, 5, 7	2.0 (1.2) 1, 1, 2, 2, 4	2.1 (1.5) 1, 1, 2, 3, 5	3.6 (1.4) 2, 2, 3, 4, 6	1.8 (1.0) 1, 1, 2, 2, 4	1.9 (1.3) 1, 1, 2, 2, 4
3	1.3 (0.7) 1, 1, 1, 2, 3	1.3 (0.6) 1, 1, 1, 1, 2	2.9 (0.8) 2, 2, 3, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.3 (0.6) 1, 1, 1, 2, 3	2.5 (0.8) 2, 2, 2, 3, 4	1.3 (0.5) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2

** indicates variance estimate is not meaningful and # indicates the percentile value exceeds 5000.

Table-7. Performance comparisons for m=100, n=5 between various competetive charts for the Exponential (θ, δ) distribution with ARL_0 = 500.

θ	Shewhart Lepage Chart	Shewhart Cucconi Chart	CUSUM Lepage chart			CUSUM Cucconi chart		
			k=0	k=3	k=6	k=0	k=1.5	k=3.0
				δ =0.50				
0	129.0 (253.0) 4, 20, 54, 137, 483	131.5 (274.7) 4, 19, 53, 135, 488	28.5 (34.8) 9 15, 22, 33, 64	104.2 (215.9) 3, 15, 42, 107, 391	124.3 (242.3) 4, 19, 53, 132, 462	27.9 (34.2) 7, 14, 21, 33, 65	118.3 (276.2) 2, 14, 41, 113, 460	133.6 (284.3) 3, 18, 52, 136, 501
0.25	*>2900 (**)* *111, 849, 3062, #, #*	*>4900 (**)* *#, #, #, #, #*	85.2 (397.6) 11, 18, 28, 48, 161	*>1700 (**)* *19, 162, 776, 3571, #*	*>2800 (**)* *99, 784, 2922, #, #*	56.7 (235.1) 15, 22, 30, 45, 109	*>4900 (**)* *#, #, #, #, #*	*>4900 (**)* *#, #, #, #, #*
0.5	*909.3 (1360.3)* *13, 90, 307, 1015, #*	*>4700 (**)* *2021, #, #, #, #*	12.5 (5.4) 7, 9, 12, 15, 21	*155.9 (485.4)* *4, 11, 31, 97, 626*	*>700 (**)* *10, 68, 235, 796, 4930*	15.9 (4.6) 10, 13, 15, 18, 24	*>4600 (**)* *1629, #,#,#,#*	*>4700 (**)* *2007, #, #, #, #*
0.75	*>1800 (**)* *23, 206, 892, 3961, #*	*>3600 (**)* *168, 1922, #, #, #*	14.0 (3.4) 8, 12, 14, 16, 19	*>1200 (**)* *7, 47, 274, 1563, #*	*>1800 (**)* *20, 184, 836, 3766, #*	13.4 (2.2) 10, 12, 13, 15, 17	*>3500 (**)* *89, 1538, #, #*	*>3700 (**)* *168, 2006, #, #, #*
1.0	*>1200 (**)* *7, 70, 335, 1652, #*	*>1900 (**)* *16, 174, 901, 4910, #*	12.7 (4.3) 6, 9, 13, 16, 20	*>900 (**)* *3, 16, 114, 919, #*	*>1200 (**)* *6, 64, 328, 1670, #*	10.6 (2.6) 6, 9, 11, 13, 14	*>1500 (**)* *4, 28, 310, 3318, #*	*>1900 (**)* *12, 161, 891, #, #*
1.5	50.8 (291.1) 1, 1, 4, 16, 154	141.9 (567.4) 1, 3, 10, 48, 556	4.6 (1.8) 3, 3, 4, 5, 8	11.1 (147.1) 1, 2, 2, 3, 10	41.1 (261.3) 1, 1, 3, 9, 119	4.8 (1.8) 2, 3, 4, 6, 8	22.1 (237.2) 1, 2, 3, 5, 18	126.4 (557.7) 1, 2, 5, 26, 485
2	1.8 (10.0) 1, 1, 1, 1, 4	4.3 (55.0) 1, 1, 1, 2, 9	2.6 (0.7) 2, 2, 3, 3, 4	1.2 (0.5) 1, 1, 1, 1, 2	1.5 (24.3) 1, 1, 1, 1, 2	2.7 (0.8) 2, 2, 2, 3, 4	1.4 (2.0) 1, 1, 1, 2, 3	2.5 (30.0) 1, 1, 1, 2, 4
3	1.0 (0.0) 1, 1, 1, 1, 1,	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.10 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.7 (0.5) 1, 1, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
				δ =1.00				
0	516.5 (686.7) 19, 106, 281, 640, 1824	486.0 (696.3) 16, 91, 245, 583, 1778	495.3 (848.1) 35, 88, 195, 486, 2063	506.8 (777.3) 23, 89, 208, 535, 2179	510 (710.3) 21, 84, 218, 555, 2231	506.7 (890.7) 27, 75, 181, 490, 2249	508.4 (757.4) 13, 87, 242, 593, 1940	499.2 (720.1) 15, 92, 248, 592, 1866
0.25	470.8 (733.8) 13, 75, 210, 536, 1835	*670.1 (1023.2)* *15, 94, 275, 746, 2960*	124.2 (420.9) 16, 28, 44, 79, 342	401.8 (646.1) 11,62,180,451,1543	467.6 (734.5) 12, 75, 209, 529, 1810	128.3 (422.1) 16, 29, 47, 83, 356	*692.4 (1073.0)* *12,87,268,760,3230*	*679.8 (1039.9)* *14,92,276,757,3053*
0.5	149.4 (278.3) 4, 23, 65, 163, 557	232.9 (474.5) 5, 30, 86, 231, 912	15.8 (7.6) 8, 12, 15, 18, 27	95.9 (176.4) 3, 15, 42, 105, 359	142.8 (261.8) 4, 22, 62, 156, 538	16.6 (7.6) 7, 12, 15, 20, 30	206.2 (467.3) 3, 20, 64, 188, 846	233.5 (485.7) 4, 28, 83, 227, 926
0.75	48.5 (95.2) 2, 7 20, 52, 184	73.3 (173.1) 2, 10, 27, 71, 278	9.6 (3.3) 5, 7, 9 12, 15	27.1 (55.5) 2, 5, 10, 27, 104	45.6 (92.7) 2, 7, 18, 47, 175	9.3 (3.6) 4, 7,9, 12, 16	44.4 (148.5) 2, 5, 12, 34, 168	68.6 (167.1) 2, 8, 24, 66, 264
1.0	15.3 (35.1) 1, 2, 6, 16, 56	22.6 (52.4) 1, 3, 9, 22, 85	6.2 (2.5) 3, 4, 6, 8, 11	6.9 (16.2) 1, 2, 3, 6, 21	13.5 (31.3) 1, 2, 5, 13, 50	6.0 (2.4) 3, 4, 6, 7, 10	8.9 (28.9) 1, 2, 4, 8, 27	19.8 (47.7) 1, 2, 7, 18, 75
1.5	2.1 (4.2) 1, 1, 1, 2, 6	3.1 (6.5) 1, 1, 1, 3, 10	3.1 (0.9) 2, 3, 3, 4, 5	1.5 (0.8) 1, 1, 1, 2, 3	1.8 (2.3) 1, 1, 1, 2, 4	3.1 (1.1) 2, 2, 3, 4, 5	1.8 (2.3) 1, 1, 2, 2, 4	2.4 (3.9) 1, 1, 1, 2, 6
2	1.0 (0.3) 1, 1, 1, 1, 1	1.1 (0.7) 1, 1, 1, 1, 2	2.2 (0.4) 2, 2, 2, 2, 3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	2.2 (0.5) 2, 2, 2, 2, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.02) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.5 (0.5) 1, 1, 1, 2, 2	1.0 (0.01) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
				δ =1.25				

0	184.9 (265.4) 7, 37, 97, 224, 652	176.6 (270.7) 6, 34, 90, 211, 618	129.2 (252.6) 17, 37, 66, 128, 416	169.7 (249.6) 6, 33, 88, 205, 604	184.2 (271.6) 7, 37, 97, 223, 639	115.5 (245.3) 13, 30, 55, 111, 374	168.1 (278.3) 4, 29, 80, 193, 617	176.5 (270.1) 5, 33, 89, 210, 627
0.25	102.4 (166.4) 4, 20, 51, 119, 366	121.1 (207.8) 4, 21, 57, 137, 442	34.8 (52.5) 10, 17, 25, 39, 83	88.1 (150.6) 3, 16, 43, 101, 313	100.3 (166.0) 4, 19, 49, 117, 356	34.2 (54.5) 8, 16, 25, 38, 82	111.7 (212.4) 2, 17, 48, 121, 416	122.0 (223.6) 3, 20, 55, 136, 446
0.5	40.8 (68.0) 2, 8, 20, 47, 144	48.8 (87.2) 2, 9, 23, 54, 175	12.1 (5.3) 5, 9, 11, 15, 21	27.8 (45.5) 2, 6, 14, 31, 98	38.7 (63.0) 2, 7, 19, 45, 138	11.9 (5.8) 4, 8, 11, 15, 22	35.1 (77.4) 2, 6, 15, 37, 130	47.0 (87.5) 2, 8, 21, 52, 172
0.75	15.6 (25.5) 1, 3, 8, 18, 55	19.1 (37.3) 1, 4, 9, 22, 68	7.4 (2.8) 4, 5, 7, 9, 13	8.9 (13.8) 1, 3, 5, 10, 29	14.7 (24.9) 1, 3, 7, 17, 52	7.0 (3.0) 3, 5, 7, 9, 13	10.4 (24.3) 1, 2, 5, 11, 34	17.5 (33.6) 1, 3, 8, 19, 63
1.0	6.1 (10.3) 1, 2, 3, 7, 20	8.0 (13.5) 1, 2, 4, 9, 28	4.9 (1.9) 3, 4, 5, 6, 8	3.4 (4.1) 1, 2, 2, 4, 9	5.4 (8.6) 1, 2, 3, 6, 18	4.8 (1.9) 2, 3, 5, 6, 8	3.9 (5.8) 1, 2, 3, 5, 10	6.7 (11.6) 1, 2, 3, 7, 23
1.5	1.5 (1.4) 1, 1, 1, 2, 4	1.9 (2.2) 1, 1, 1, 2, 5	2.8 (0.8) 2, 2, 3, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.4 (1.1) 1, 1, 1, 2, 3	2.8 (0.9) 2, 2, 3, 3, 4	1.5 (0.8) 1, 1, 1, 2, 3	1.7 (1.5) 1, 1, 1, 2, 4
2	1.0 (0.2) 1, 1, 1, 1, 1	1.1 (0.4) 1, 1, 1, 1, 1	2.1 (0.3) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	2.1 (0.5) 1, 2, 2, 2, 3	1.1 (0.2) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.01) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.4 (0.5) 1, 1, 1, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =1.50								
0	63.4 (86.2) 3, 14, 36, 79, 213	61.4 (86.3) 3, 13, 34, 75, 210	39.1 (42.1) 10, 18 29, 46, 99	55.7 (81.1) 3, 12, 30, 67, 189	63.0 (86.3) 3 14, 35, 78, 214	33.0 (41.8) 7, 15, 24, 39, 85	52.7 (81.5) 2, 10, 27, 63, 186	60.3 (88.1) 2, 12, 32, 73, 210
0.25	35.8 (50.7) 2, 8, 20, 44, 121	38.5 (56.2) 2, 9, 21, 47, 132	18.4 (11.9) 6, 11, 16, 23, 40	28.6 (41.0) 2, 7, 16, 34, 97	34.6 (48.2) 2, 8, 19, 42, 118	17.3 (12.0) 5, 10, 15, 22, 38	31.4 (51.1) 2, 6, 16, 37, 112	37.3 (56.1) 2, 8, 20, 45, 129
0.5	16.7 (22.7) 1, 4, 9, 21, 56	18.2 (25.7) 1, 4, 10, 22, 61	9.3 (4.1) 4, 6, 9, 11, 17	11.5 (15.2) 1, 3, 7, 14, 37	15.7 (21.5) 1, 4, 9, 19, 53	8.8 (4.3) 3, 6, 8, 11, 16	12.1 (18.5) 1, 3, 7, 14, 40	17.0 (25.7) 1, 3, 9, 21, 58
0.75	7.7 (10.3) 1, 2, 4, 9, 25	8.8 (12.0) 1, 2, 5, 11, 29	6.0 (2.3) 3, 4, 6, 7, 10	4.8 (5.4) 1, 2, 3, 6, 14	7.0 (9.4) 1, 2, 4, 8, 23	5.6 (2.5) 2, 4, 5, 7, 10	5.0 (6.0) 1, 2, 3, 6, 14	7.7 (11.3) 1, 2, 4, 9, 26
1.0	3.6 (4.5) 1, 1, 2, 4, 11	4.4 (5.6) 1, 1, 3, 5, 14	4.2 (1.5) 2, 3, 4, 5, 7	2.4 (1.9) 1, 1, 2, 3, 6	3.2 (3.8) 1, 1, 2, 4, 10	4.0 (1.6) 2, 3, 4, 5, 7	2.7 (2.2) 1, 1, 2, 3, 6	3.6 (4.5) 1, 1, 2, 4, 11
1.5	1.3 (0.8) 1, 1, 1, 1, 3	1.5 (1.3) 1, 1, 1, 2, 4	2.6 (0.7) 2, 2, 2, 3, 4	1.2 (0.5) 1, 1, 1, 1, 2	1.2 (0.7) 1, 1, 1, 1, 2	2.5 (0.8) 2, 2, 2, 3, 4	1.3 (0.6) 1, 1, 1, 2, 2	1.4 (0.9) 1, 1, 1, 2, 3
2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	2.1 (0.3) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.5) 1, 2, 2, 2, 3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.3 (0.5) 1, 1, 1, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =1.75								
0	28.6 (36.3) 2, 7, 17, 36, 93	27.1 (34.8) 2, 7, 16, 34, 89	20.2 (13.7) 6, 11, 17, 25, 45	23.5 (30.1) 2, 6, 14, 30, 77	28.0 (34.7) 2, 7, 17, 36, 92	17.0 (12.2) 4, 9, 14, 21, 39	21.5 (29.1) 2, 5, 12, 27, 73	26.0 (35.2) 2, 6, 15, 33, 87
0.25	17.3 (21.4) 1, 4, 10, 22, 56	17.9 (22.8) 1, 4, 11, 23, 58	12.4 (6.8) 5, 8, 11, 15, 25	13.4 (16.9) 1, 4, 8, 17, 43	16.5 (20.7) 1, 4, 10, 21, 54	11.4 (6.8) 3, 7, 10, 14, 24	13.1 (17.5) 1, 3, 8, 16, 43	17.0 (22.8) 1, 4, 10, 21, 56
0.5	9.3 (11.3) 1, 3, 6, 12, 30	9.7 (12.3) 1, 3, 6, 12, 31	7.4 (3.1) 3, 5, 7, 9, 13	6.5 (7.1) 1, 2, 4, 8, 19	8.8 (10.6) 1, 2, 5, 11, 28	6.8 (3.3) 2, 4, 6, 9, 13	6.4 (7.7) 1, 2, 4, 8, 19	8.7 (11.1) 1, 2, 5, 11, 29
0.75	4.8 (5.5) 1, 1, 3, 6, 15	5.2 (6.1) 1, 2, 3, 6, 16	5.0 (1.9) 3, 4, 5, 6, 9	3.3 (3.1) 1, 2, 2, 4, 9	4.5 (5.1) 1, 1, 3, 5, 14	4.7 (2.0) 2, 3, 4, 6, 8	3.4 (3.1) 1, 2, 2, 4, 9	4.6 (5.4) 1, 2, 3, 6, 14
1.0	2.6 (2.7) 1, 1, 2, 3, 7	3.0 (3.1) 1, 1, 2, 4, 9	3.7 (1.2) 2, 3, 3, 4, 6	2.0 (1.3) 1, 1, 2, 2, 4	2.4 (2.3) 1, 1, 2, 3, 6	3.5 (1.4) 2, 2, 3, 4, 6	2.1 (1.5) 1, 1, 2, 2, 5	2.6 (2.6) 1, 1, 2, 3, 7
1.5	1.2 (0.6) 1, 1, 1, 1, 2	1.3 (0.8) 1, 1, 1, 1, 3	2.4 (0.6) 2, 2, 2, 3, 3	1.1 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2	2.4 (0.7) 2, 2, 2, 3, 4	1.2 (0.5) 1, 1, 1, 1, 2	1.3 (0.6) 1, 1, 1, 1, 2
2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	2.0 (0.2) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.9 (0.5) 1, 2, 2, 2, 3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.3 (0.4) 1, 1, 1, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =2.00								

0	16.1 (119.0) 1, 4, 10, 21, 51	15.0 (17.9) 1, 4, 9, 19, 48	13.4 (7.7) 5, 8, 12, 17, 28	12.8 (14.9) 1, 4, 8, 16, 40	15.3 (17.8) 1, 4, 10, 20, 49	11.4 (7.0) 3, 7, 10, 15, 24	11.3 (13.9) 1, 3, 7, 14, 36	14.0 (16.9) 1, 3, 8, 18, 45
0.25	10.3 (11.7) 1, 3, 6, 13, 32	10.4 (12.1) 1, 3, 7, 13, 33	9.2 (4.6) 4, 6, 8, 11, 18	7.9 (8.7) 1, 3, 5, 10, 24	9.9 (11.2) 1, 3, 6, 13, 31	8.3 (4.6) 2, 5, 8, 11, 17	7.5 (8.6) 1, 2, 5, 9, 23	9.6 (11.4) 1, 2, 6, 12, 31
0.5	6.1 (6.8) 1, 2, 4, 8, 19	6.2 (6.8) 1, 2, 4, 8, 19	6.1 (2.5) 3, 4, 6, 7, 11	4.4 (4.2) 1, 2, 3, 6, 12	5.8 (6.4) 1, 2, 4, 7, 17	5.6 (2.7) 2, 4, 5, 7, 10	4.3 (4.2) 1, 2, 3, 5, 12	5.5 (6.4) 1, 2, 3, 7, 17
0.75	3.5 (3.6) 1, 1, 2, 4, 10	3.7 (3.8) 1, 1, 2, 5, 11	4.4 (1.6) 2, 3, 4, 5, 7	2.6 (2.0) 1, 1, 2, 3, 6	3.2 (3.2) 1, 1, 2, 4, 9	4.1 (1.7) 2, 3, 4, 5, 7	2.7 (2.1) 1, 1, 2, 3, 6	3.2 (3.3) 1, 1, 2, 4, 9
1.0	2.1 (1.8) 1, 1, 1, 2, 5	2.3 (2.1) 1, 1, 2, 3, 6	3.3 (1.1) 2, 3, 3, 4, 5	1.7 (1.0) 1, 1, 1, 2, 4	1.9 (1.6) 1, 1, 1, 2, 5	3.2 (1.2) 2, 2, 3, 4, 5	1.8 (0.4) 1, 1, 1, 1, 2	2.1 (1.7) 1, 1, 2, 2, 5
1.5	1.1 (0.5) 1, 1, 1, 1, 2)	1.2 (0.6) 1, 1, 1, 1, 2	2.3 (0.5) 2, 2, 2, 3, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	2.3 (0.6) 1, 2, 2, 3, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
2	1.0 (0.1) 1 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.0 (0.2) 2, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.8 (0.5) 1, 2, 2, 2, 2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1 1	1.2 (0.4) 1, 1, 1, 1, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1

** indicates unstable variance estimate and # indicates the percentile value exceeds 5000.

Table-8. Performance comparisons for m=300, n=5 between various competetive charts for the Exponential (θ,δ) distribution with ARL_0 = 500.

θ	Shewhart Lepage Chart	Shewhart Cucconi Chart	CUSUM Lepage chart			CUSUM Cucconi chart		
			k=0	*k*=3	*k*=6	*k*=0	*k*=1.5	*k*=3.0
				δ =0.50				
0	81.9 (102.7) 4, 20, 49, 104, 272	98.2 (130.2) 4, 23, 56, 124,329	27.9 (13.7) 12, 18, 25, 34, 53	83.4 (113.5) 4, 19, 47, 103, 280	104.1 (136.9) 5, 24, 61, 132, 346	28.2 (15.1) 10, 18, 25, 35, 56	76.5 (108.0) 2, 16, 42, 94, 264	94.3 (125.3) 3, 21, 54, 119, 322
0.25	*>3600 (**)* *321, 1972, #, #, #*	*>4900 (**)* *#, #, #, #, #*	40.3 (44.4) 16, 24, 33, 46, 84	*>2400 (**)* *74, 526, 1850, #, #*	*>4400 (**)* *940, #, #, #, #*	42.3 (20.5) 22, 30, 38, 49, 77	*>4900 (**)* *#, #, #, #, #*	*>4900 (**)* *#, #, #, #, #*
0.5	*989.3 (1252.8)* *28, 170, 487, 1244, 4414*	*>4800 (**)* *#, #, #, #, #*	13.1 (3.4) 8, 11, 13, 15, 19	*66.3 (137.3)* *5, 13, 30, 68, 234*	*>1300 (**)* *35, 220, 654, 1854, #*	19.5 (3.7) 14, 17, 19, 22, 26	*>4800 (**)* *#, #, #, #, #*	*>4800 (**)* *#, #, #, #, #*
0.75	*>2600 (**)* *97, 688, 2281, #, #*	*>4000 (**)* *488, 3456, #, #, #*	17.6 (3.4) 12, 15, 18, 20, 23	*>2100 (**)* *26, 282, 1413, #, #*	*>3200 (**)* *167, 1245, 4275, #, #*	17.6 (2.0) 14, 16, 18, 19, 21	*>4000 (**)* *389, 3243, #, #, #*	*>4000 (**)* *487, 3537, #, #, #*
1.0	*917.0 (1304.8)* *17, 111, 358, 1067, 4914*	*>1800 (**)* *42, 296, 991, 3117, #*	15.0 (4.1) 9, 12, 15, 18, 22	*>700 (**)* *6, 31, 150, 747, #*	*>1300 (**)* *23, 167, 582, 1885, #*	13.5 (2.6) 9, 12, 14, 15, 17	*>1100 (**)* *8, 50, 268, 1429, #*	*>1800 (**)* *36, 282, 986, 3161, #*
1.5	11.6 (28.5) 1, 2, 4, 11, 44	36.1 (114.4) 1, 4, 11, 31, 138	4.8 (1.1) 3, 4, 5, 5, 7	2.5 (1.5) 1, 2, 2, 3, 5	8.3 (38.1) 1, 2, 3, 6, 25	5.3 (1.4) 3, 4, 5, 6, 8	3.3 (3.6) 1, 2, 3, 4, 7	17.7 (76.1) 1, 2, 4, 11, 63
2	1.1 (0.5) 1, 1, 1, 1, 2	1.4 (1.7) 1, 1, 1, 1, 3	2.9 (0.5) 2, 3, 3, 3, 4	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.9 (0.6) 2, 2, 3, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.5) 1, 1, 1, 1, 2
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.1) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
				δ =1.00				
0	501.5 (556.7) 23,130,322,673,1577	501.5 (585.4) 22,125,309,662,1621	484.3 (691.8) 54,131,261,532,1487	503.1 (613.9) 32,109,342,841,1848	500.9 (593.1) 21,101,409,938,2233	504.7 (717.3) 48,124,258,564,1825	501.7 (601.2) 18,117,302,655,1664	504.6 (589.3) 21,124,310,662,1649
0.25	396.4 (507.1) 16, 91, 230, 505, 1323	*527.3 (679.1)* *20,115,297,665,1823*	66.2 (70.4) 22, 36, 51, 75, 150	494.3 (631.8) 20,110,278,624,1699	*540.3 (680.5)* *21, 123, 309, 689*	75.4 (85.2) 24, 41, 59, 86, 170	*529.8 (704.9)* *16,108,288,660,1888*	*528.4 (693.5)* *18,112,292,659,1872*
0.5	117.4 (152.4) 5, 27, 68, 150, 392	162.3 (221.7) 7, 36, 91, 202, 548	17.2 (4.9) 10, 14, 17, 20, 26	97.3 (127.7) 5, 22, 55, 123, 328	155.8 (210.9) 6, 34, 88, 194, 526	19.4 (6.2) 10, 15, 19, 23, 30	125.6 (192.1) 4, 24, 64, 150, 449	159.3 (226.1) 5, 33, 86, 197, 553
0.75	34.9 (46.1) 2, 8, 20, 44, 117	49.2 (67.1) 2, 11, 28, 61, 165	10.8 (3.1) 6, 9, 11, 13, 16	20.3 (28.2) 2, 6, 11, 24, 67	42.1 (57.2) 2, 9, 24, 52, 143	10.8 (3.4) 6, 8, 11, 13, 17	21.8 (33.6) 2, 5, 12, 25, 73	44.4 (65.6) 2, 9, 24, 54, 152

1.0	10.6 (13.5) 1, 3, 6, 13, 34	15.6 (20.4) 1, 4, 9, 19, 51	6.7 (2.0) 4, 5, 6, 8, 10	4.8 (4.4) 1, 2, 4, 6, 12	10.6 (14.7) 1, 3, 6, 13, 35	6.8 (2.2) 3, 5, 7, 8, 11	5.3 (5.0) 1, 2, 4, 7, 14	11.9 (16.9) 1, 3, 6, 14, 40
1.5	1.6 (1.3) 1, 1, 1, 2, 4	2.3 (2.2) 1, 1, 1, 3, 6	3.4 (0.7) 2, 3, 3, 4, 5	1.5 (0.6) 1, 1, 1, 2, 2	1.6 (0.9) 1, 1, 1, 2, 3	3.4 (0.9) 2, 3, 3, 4, 5	1.7 (0.7) 1, 1, 2, 2, 3	1.8 (1.2) 1, 1, 1, 2, 4
2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	2.4 (0.5) 2, 2, 2, 3, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.3 (0.5) 2, 2, 2, 3, 3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1
3	1.0 (0.00 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.9 (0.2) 1, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =1.25								
0	169.4 (195.2) 8, 44, 107, 223, 542	164.4 (193.1) 8, 41, 102, 216, 530	104.3 (98.6) 25, 48, 77, 126, 272	211.9 (255.8) 10, 52, 129, 276, 695	223.4 (261.2) 10, 57, 139, 294, 719	91.8 (93.2) 20, 41, 67, 110, 242	148.0 (180.1) 5, 35, 90, 193, 485	162.1 (193.4) 6, 40, 100, 213, 526
0.25	84.8 (100.6) 4, 22, 53, 111, 272	97.7 (117.7) 5, 24, 60, 126, 317	32.5 (17.3) 13, 21, 29, 40, 64	91.5 (111.2) 5, 23, 56, 118, 297	107.6 (129.3) 5, 27, 66, 140, 346	33.0 (18.2) 12, 21, 29, 41, 67	82.5 (105.7) 3, 19, 48, 106, 276	94.2 (116.3) 3, 22, 56, 123, 309
0.5	32.8 (38.1) 2, 9, 21, 43, 105	39.1 (46.3) 2, 10, 24, 51, 126	13.1 (4.3) 7, 10, 13, 16, 21	25.2 (30.2) 2, 7, 16, 32, 80	39.3 (46.7) 2, 10, 24, 51, 126	13.5 (5.1) 6, 10, 13 17, 23	24.2 (30.7) 2, 6, 14, 30, 80	35.6 (44.5) 2, 8, 21, 46, 117
0.75	12.5 (14.3) 1, 3, 8, 16, 39	15.4 (18.0) 1, 4, 10, 20, 49	8.1 (2.5) 4, 6, 8, 10, 13	7.4 (7.3) 1, 3, 5, 9, 20	13.6 (15.9) 1, 4, 8, 17, 43	8.0 (2.8) 4, 6, 8, 10, 13	7.2 (7.5) 1, 2, 5, 9, 20	12.9 (15.5) 1, 3, 8, 17, 42
1.0	4.9 (5.2) 1, 2, 3, 6, 15	6.4 (7.2) 1, 2, 4, 8, 20	5.3 (1.5) 3, 4, 5, 6, 8	3.0 (1.9) 1, 2, 3, 4, 7	4.7 (4.9) 1, 2, 3, 6, 14	5.3 (1.8) 3, 4, 5, 6, 8	3.2 (2.2) 1, 2, 3, 4, 7	4.9 (5.3) 1, 2, 3, 6, 14
1.5	1.3 (0.8) 1, 1, 1, 1, 3	1.6 (1.2) 1, 1, 1, 2, 4	3.1 (0.6) 2, 3, 3, 3, 4	1.3 (0.5) 1, 1, 1, 2, 2	1.3 (0.6) 1, 1, 1, 2, 2	3.0 (0.8) 2, 2, 3, 4, 4	1.4 (0.6) 1, 1, 1, 2, 2	1.5 (0.8) 1, 1, 1, 2, 3
2	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.2) 1, 1, 1, 1, 1	2.2 (0.4) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.2 (0.4) 2, 2, 2, 2, 3	1.0 (0.2) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	1.9 (0.3) 1, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =1.50								
0	58.5 (64.4) 3, 16, 38, 79, 183	55.6 (61.6) 3, 15, 36, 74, 175	37.9 (22.9) 13, 22, 33, 47, 80	61.9 (71.3) 4, 16, 39, 81, 196	71.9 (81.3) 4, 19, 46, 95, 228	32.9 (20.7) 10, 19, 28, 41, 71	44.5 (52.3) 2, 11, 28, 59, 143	53.1 (61.1) 2, 13, 33, 71, 170
0.25	31.6 (34.7) 2, 9, 21, 42, 98	33.5 (37.4) 2, 9, 22, 45, 105	19.3 (8.8) 8, 13, 18, 24, 36	29.2 (32.8) 2, 8, 19, 38, 91	37.4 (42.5) 2, 10, 24, 50, 117	18.6 (9.2) 6, 12, 17, 23, 35	24.8 (28.8) 2, 6, 16, 32, 80	31.3 (36.2) 2, 8, 20, 42, 100
0.5	14.7 (15.7) 1, 4, 10, 19, 45)	16.0 (17.7) 1, 5, 10, 21, 50	10.1 (3.5) 5, 8, 10, 12, 16	10.7 (10.9) 1, 4, 7, 14, 31	16.5 (18.2) 1, 5, 11, 22, 51	9.8 (3.9) 4, 7, 9, 12, 17	9.5 (10.0) 1, 3, 6, 12, 28	14.0 (15.9) 1, 4, 9, 19, 44
0.75	6.6 (6.8) 1, 2, 4, 9, 20	7.7 (8.2) 1, 2, 5, 10, 23	6.5 (2.1) 4, 5, 6, 8, 10	4.3 (3.4) 1, 2, 3, 6, 11	6.8 (7.2) 1, 2, 5, 9, 20	6.3 (2.3) 3, 5, 6, 8, 10	4.2 (3.4) 1, 2, 3, 6, 11	6.3 (6.7) 1, 2, 4, 8, 19
1.0	3.1 (2.9) 1, 1, 2, 4, 9	3.8 (3.7) 1, 1, 3, 5, 11	4.5 (1.2) 3, 4, 4, 5, 7	2.3 (1.3) 1, 1, 2, 3, 5	3.0 (2.6) 1, 1, 2, 4, 8	4.5 (1.5) 2, 3, 4, 5, 7	2.5 (1.5) 1, 1, 2, 3, 5	3.0 (2.7) 1, 1, 2, 4, 8
1.5	1.2 (0.6) 1, 1, 1, 1, 2	1.4 (0.8) 1, 1, 1, 1, 3	2.8 (0.6) 2, 2, 3, 3, 4	1.2 (0.4) 1, 1, 1, , 2	1.2 (0.5) 1, 1, 1, 1, 2	2.8 (0.7) 2, 2, 3, 3, 4	1.3 (0.5) 1, 1, 1, 2, 2	1.3 (0.6) 1, 1, 1, 1, 2
2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.1 (0.3) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.1 (0.4) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.8 (0.3) 1, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =1.75								
0	27.0 (28.6) 2, 8, 18, 36, 83	25.4 (27.4) 2, 7, 17, 34, 79	21.3 (10.8) 8, 14, 19, 27, 42	25.5 (27.4) 2, 7, 17, 34, 78	31.4 (33.7) 2, 9, 21, 42, 97	18.4 (9.9) 6, 12, 17, 23, 37	18.6 (20.3) 2, 5, 12, 25, 58	23.3 (25.8) 2, 6, 15, 31, 73
0.25	15.9 (16.6) 1, 5, 11, 21, 48	16.2 (17.0) 1, 5, 11, 22, 49	13.3 (5.7) 6, 9, 12, 16, 24	13.5 (13.8) 2, 4, 9, 18, 40	17.9 (19.2) 1, 5, 12, 24, 55	12.4 (5.9) 4, 8, 12, 16, 23	11.3 (12.0) 1, 3, 7, 15, 34	14.6 (16.0) 1, 4, 9, 19, 46
0.5	8.4 (8.6) 1, 3, 6, 11, 25	8.9 (9.2) 1, 3, 6, 12, 27	8.0 (2.8) 4, 6, 8, 10, 13	6.2 (5.5) 1, 3, 5, 8, 17	9.0 (9.2) 1, 3, 6, 12, 27	7.6 (3.2) 3, 5, 7, 10, 13	5.5 (5.0) 1, 2, 4, 7, 15	7.6 (7.9) 1, 2, 5, 10, 23
0.75	4.3 (4.1) 1, 2, 3, 6, 12	4.8 (4.7) 1, 2, 3, 6, 14	5.5 (1.7) 3, 4, 5, 6, 9	3.2 (2.2) 1, 2, 3, 4, 7	4.3 (4.0) 1, 2, 3, 6, 12	5.2 (2.0) 2, 4, 5, 6, 9	3.1 (2.2) 1, 2, 2, 4, 7	3.9 (3.8) 1, 2, 3, 5, 11

1.0	2.3 (1.9) 1, 1, 2, 3, 6	2.7 (2.4) 1, 1, 2, 3, 7	4.0 (1.1) 3, 3, 4, 5, 6	2.0 (1.0) 1, 1, 2, 2, 4	2.3 (1.7) 1, 1, 2, 3, 6	3.9 (1.3) 2, 3, 4, 5, 6	2.0 (1.1) 1, 1, 2, 2, 4	2.3 (1.7) 1, 1, 2, 3, 6
1.5	1.1 (0.4) 1, 1, 1, 1, 2	1.2 (0.6) 1, 1, 1, 1, 2	2.7 (0.6) 2, 2, 3, 3, 3	1.2 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	2.6 (0.7) 2, 2, 2, 3, 4	1.2 (0.4) 1, 1, 1, 1, 2	1.2 (0.4) 1, 1, 1, 1, 2
2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1	2.1 (0.3) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.1 (0.3) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.0) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.8 (0.4) 1, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1
δ =2.00								
0	15.1 (15.5) 1, 4, 10, 20, 45	14.1 (14.4) 1, 4, 9, 19, 43	14.5 (6.7) 6, 10, 13, 18, 27	13.4 (13.5) 1, 4, 9, 18, 40	17.0 (17.6) 1, 5, 11, 23, 51	12.5 (6.4) 4, 8, 12, 16, 24	10.2 (10.3) 1, 3, 7, 14, 30	12.8 (13.5) 1, 4, 9, 17, 39
0.25	9.7 (9.80 1, 3, 7, 13, 29	9.7 (9.8) 1, 3, 7, 13, 29	10.1 (4.1) 4, 7, 9, 12, 18	8.1 (7.6) 1, 3, 6, 11, 23	10.6 (10.7) 1, 3, 7, 14, 31	9.3 (4.3) 3, 6, 9, 12, 17	6.7 (6.4) 1, 2, 5, 9, 19	8.6 (8.8) 1, 2, 6, 11, 26
0.5	5.7 (5.50 1, 2, 4, 8, 16	5.7 (5.6) 1, 2, 4, 8, 17	6.7 (2.3) 3, 5, 6, 8, 11	4.4 (3.5) 1, 2, 3, 6, 11	5.9 (5.7) 1, 2, 4, 8, 17	6.2 (2.6) 2, 4, 6, 8, 11	3.9 (3.1) 1, 2, 3, 5, 10	4.9 (4.8) 1, 2, 3, 6, 14
0.75	3.2 (2.9) 1, 1, 2, 4, 9	3.4 (3.1) 1, 1, 2, 4, 9	4.8 (1.5) 3, 4, 5, 6, 7	2.6 (1.6) 1, 1, 2, 3, 6	3.2 (2.7) 1, 1, 2, 4, 9	4.5 (1.7) 2, 3, 4, 6, 8	2.5 (1.6) 1, 1, 2, 3, 6	2.9 (2.4) 1, 1, 2, 4, 8
1.0	1.9 (1.4) 1, 1, 1, 2, 5	2.1 (1.7) 1, 1, 2, 3, 5	3.6 (1.0) 2, 3, 4, 4, 5	1.8 (0.9) 1, 1, 2, 2, 3	1.9 (1.3) 1, 1, 2, 2, 4	3.5 (1.2) 2, 3, 3, 4, 6	1.8 (0.9) 1, 1, 2, 2, 4	1.9 (1.2) 1, 1, 2, 2, 4
1.5	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2	2.5 (0.5) 2, 2, 3, 3, 3	1.1 (0.3) 1, 1, 1, 1, 2	1.1 (0.3) 1, 1, 1, 1, 2	2.5 (0.6) 2, 2, 2, 3, 4	1.1 (0.4) 1, 1, 1, 1, 2	1.1 (0.4) 1, 1, 1, 1, 2
2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.2) 2, 2, 2, 2, 3	1.0 (0.04) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.3) 2, 2, 2, 2, 3	1.0 (0.1) 1, 1, 1, 1, 1	1.0 (0.1) 1, 1, 1, 1, 1
3	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	2.0 (0.2) 2, 2, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1, 1	1.0 (0.0) 1, 1, 1, 1, 1	1.7 (0.4) 1, 1, 2, 2, 2	1.0 (0.0) 1, 1, 1, 1 1	1.0 (0.0) 1, 1, 1, 1, 1

** indicates variance estimate is not meaningful and # indicates the percentile value exceeds 50

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Abstract:

Recently, Chowdhury et al. (2014a) proposed a single distribution-free Shewhart-type control chart based on the Cucconi (1968) test statistic for monitoring shift in the unknown location and scale parameters of a process distribution simultaneously. Several recent researches demonstrated that the CUSUM type charts perform better than the Shewhart-type charts under small and persistent shift. In the present work, we develop a phase II distribution-free CUSUM chart based on the Cucconi statistic, referred to as CUSUM-Cucconi (CC) chart. Nonparametric nature of the Cucconi statistic ensures that all the in control (IC) properties of the proposed chart remain invariant and known for all continuous process distributions. Control limits are tabulated for implementation of the chart. The IC and out of control (OOC) performance of the chart are thoroughly investigated in terms of the average, standard deviation, median and some percentiles of the corresponding run length distributions. A detailed comparison with the Shewhart-type Cucconi and Lepage charts as well as the CUSUM Lepage chart (as in Chowdhury et al. (2014b)) is presented. The proposed chart is illustrated with exchange rates data.

Key Words/Phrases: Cucconi Statistic; Average Run Length; Upper Control Limit; CUSUM Cucconi Chart; Nonparametric; Monte-Carlo Simulation; Statistical Process Control.

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