Agent Based modeling of Housing Asset Bubble: A Simple Utility Function Based Investigation

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Abstract The housing asset bubble and mortgage crisis of 2007-08 in the US market poses a challenge to understanding of market and hypotheses related to market efficiency. The contribution of our paper is bifold. First, we present a survey of the existing literature which explains the housing asset bubble. We have emphasized on agent based modeling approaches in this context. The second part of the paper frames an economic model to demonstrate the power of irrational "exuberance hypothesis", a term coined by Robert J Shiller. Using a felicity function based framework, this shows that the power of irrational expectation in bringing about an artificial and unintended boost in demand for investment of housing asset.

1 Introduction

The world at large was at a loss to explain the magnitude as well as nature of calamity that hit the US market in 2007. Economic thoughts are being re-organised and re-structured even now in search of a definite analytical framework to explain the failure of what was thought to be a fail-proof wealth generating system. This unprecedented crisis in the financial market engendered theories on financial markets which eventually adds to refinement of economic thinking. The qualitative way of thinking can point out to factors relating to human behaviour and

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its departure from presumed economic rationality regarding decision-making. However, any qualitative story should be supplemented with sufficient quantitative illustration for general acceptance. A quantitative model of an underlying qualitative story provides us with the power of the story to explain this phenomenon.

What is the root underlying cause of this financial crisis? Is it engendered by "bad" but otherwise improbable run of flawed decisions, or is there any fundamental flaw in the financial system? Hellwig [2] argues that the cause of this crisis was not embedded in some flawed decisions rather in financial system architecture. The International Monetary Fund (IMF) forecasted a total loss of 750 billion dollars [3] in US residential real-estate lending, by October 2008. This amount, though a large sum in the housing market, is relatively low if analysed in the context of the size of global financial system. Moreover, the decline in the value of financial securities is far more compared to the fall is housing prices. These indicate deeper flaw in the financial system than a mere coincidental fall in housing prices.

The first part of Hellwig's work [2] discusses the "financial system architecture". More specifically, the mechanisms of risk management are discussed. One example is the process of mortgage securitization. It acts based on the principle of diversification to mitigate risks from interest rate fluctuations. The problems in the financial system architecture was traced back to basic theories in economics of information – "moral hazard" and "adverse selection". The paper cites one specific example to illustrate that sellers used to display higher prices in the contract and the additional money used to be given back to buyers as advance payment. This system takes risk away from buyers, one of the stakeholders in a risky investment of purchasing a house, to create moral hazard problem. The other related failures from different stakeholders such as, Rating Agency, internal correction, market discipline, has also been discussed.

The contribution of incidence of systemic risk is analysed in the second part of Hellwig's work [2]. There could be various ways to understand the incidence of systematic risk and its augmentation. The perennial problem is transformation of long term investment to short term investment which is done through conduits and structured-investment vehicles (SIVs). There is some systemic risk involved in this transformation. At the onset of this financial crisisin August 2007, an excessive amount of assets are transformed in this manner compared to historical average. This excessive supply actually plummeted down security prices considerably. At the instance of public recognition of delinquencies and defaults, there is a host of factors which led to market breakdown. There factors are identified as lack of fair value accounting and the insufficiency of equity capital at financial institutions.

Many theoretical and empirical works have come out to explain the phenomenon particularly the housing asset bubble that started it all. We surveyed the present literature in sufficient detail to present a thorough understanding to reader. The present literature may not be conclusive enough. We draw key insights from this literature, and then present our model based on our insights. The organization of this paper follows here. Section 2 presents an elaborate survey of the present literature. Section 3 states our economic environment and its contribution to demonstrate the power of "irrational exuberance" in defining expectation. Section 4 discusses our contribution to this literature.

2 Existing Literature: Sub Prime Crisis and Agent Based Modelling

Wray [4] (also, [5]) use Hyman P. Minsky's approach to analyze the international financial crisis initiated by problems in the U.S. real estate market. They examine the role played by each of the key players-including brokers, appraisers, borrowers, securitizers, insurers, and regulators-in creating the crisis.This paper uses Hyman P. Minsky's approach to analyze the current international financial crisis that was initiated by problems in the U.S. real estate market. In a 1987 manuscript, Minsky had already recognized the importance of the trend toward securitization of home mortgages. This paper identifies the causes and consequences of the financial innovations that created the real estate boom and bust. It examines the role played by each of the key players including brokers, appraisers, borrowers, securitizers, insurers, and regulators in creating the crisis. Finally, it proposes short-run solutions to the current crisis, as well as longer-run policy measures to prevent a debt deflation from happening again.

Goodman and Thibodeau [6], takes a call on the housing asset bubble in the US market from the perspective of economic fundamentals.

They investigate how much of the price rise was caused by the demand supply dynamics of the housing economy. The exorbitant rise of housing price index in the early years of the first decade of the new millennium led many economists to postulate theories of speculative bubbles and herd behaviour. Such a conclusion can only be confirmed if the rise in price cannot be explained by the fundamentals of the governing economy. The authors worked on this premise and examined the demand supply dynamics of the housing market in the US. They examines what was the relative contribution of the fundamentals and the speculative phenomenon to the price rise. They have approached this problem in a two pronged manner. At first, they have used a simulation based model of the housing price behaviour in the long run. After that they have done empirical investigation of 133 metropolitan areas across the USA testing for the elasticity of supply in the housing market in these areas.

While working on the simulation, they [6] examine the 'shift in aggregate demand' necessary to result in the 10.3% rise in home ownership. They also verify whether the price rise resulted from a shift in equilibrium. They have simulated the demand for various elasticities of supply against a constant elasticity of demand. Their findings indicate that the price rise is extremely sensitive to the elasticity of supply. For their empirical work, the authors work with a "long-run equilibrium housing market model" which is able to explain the price variation of housing assets across the areas. Their analysis has resulted in positive elasticity of supply in 84 Metropolitan Statistical Areas (MSA) during the no-bubble period of 1990–2000. Then, their analysis has been extended to the bubble period. They have accounted for the change in fundamentals and estimated the expected price rise from the model. Results show that the speculative increase in housing asset prices was an extremely localised phenomenon as opposed the general impression of a market-wide speculative bubble. As a benchmark, the authors have considered 30% over the expected increase as a housing bubble threshold.

Coleman et al. [7] show that the subprime crisis was more of a systemic issue arising out of complex interaction of multiple agents of the financial system making it rather a "joint product" of the institutional, political and regulatory framework prevailing at the time of the crisis. This is an alternative perspective to the one that prescribes the speculative pricing of housing assets as the main cause of the crisis.

Their study reveals that the crisis cannot be ascribed to the existence of sub prime market alone. They use a simple model which is based on the change in loan intensity being leading indicator of future home prices. They have considered quantity of housing demanded in period *t* as the dependent variable. They model the movement of this dependent variable using several independent variables such as, housing prices at time *t*, vector of loan type intensity lagged, vector of macroeconomic, demographic, and financial controls, cost of capital, quantity of housing supplied in period *t*, housing market supply regulation and cost to supply housing. Subsequently, the authors impose the demand supply equilibrium conditions allowing the market imperfections to get corrected over time. A pooled cross-sectional time series was constructed which included 20 metropolitan areas for 36 quarters during the period 1998–2006.

One of the most striking findings of their initial model is that there was almost no significance of the proportion of sub prime loans among all loan in explaining the future housing price. These results also show that the increase in proportion of sub prime loans on the other hand had a positive correlation with past returns on housing prices indicating a strong dependence. This was a reinforcement of their argument that the intensity of sub prime loans did not play a causal role in the run-up of housing prices. The main macroeconomic variables which were identified as the drivers of housing price movement are aggregate level of mortgage lending, population growth, and the unemployment rate. Using "supply constraint" index as a proxy for regulatory policy, they have found a positive relationship between this variable-in-question and the housing price. The supply price was found to be significant in explaining the price movement only in case of high price assets while its significance was not present in case of middle and low value assets The authors [7] use a second model to investigate the effect of shift of the role of dominant players from the Government agencies like Freddie Mac and Fannie Mae to the private players on the dynamics of housing market. For the data set belonging to the period before the shift, the macroeconomic fundamentals could explain the price significantly, while their explanatory power was not significant in case of the data set belonging to post-shift period. Overall, it may be concluded that this study absolved the sub prime loan products of their role as the prime accused in the crisis. The whole affair was phenomenological in nature with the entire multi body system contributing to the problem in a complex manner.

Crouhyet al. [8], examine the various factors that have led to the subprime mortgage credit crisis. They identify the following factors as the main causes for the failure: yield enhancement, investment management, agency problems, lax underwriting standards, rating agency incentive problems, poor risk management by financial institutions, the lack of market transparency, the limitation of extant valuation models, the complexity of financial instruments, and the failure of regulators to understand the implications of the changing environment for the financial system. Looking at the chronological development of the problem the authors have examined the evolution of the crisis and analysed the factors that the crisis may be attributed to. The final picture that emerges in their analysis is a multi-agent complex phenomenon. The several causes that the authors point out are described here to justify the above statement. Firstly, the interest rate being low there was a need for "yield enhancement". Then there was an automatic demand for asset pooling to take benefit of financial engineering by dumping the high yield assets into the collateral pool. For this an automatic choice was sub prime loans along with auto loan and credit cards. Securitsation meant that mortgage originators were out of "default risk" and therefore they had no reason to perform due diligence. This was coupled with relaxed regulatory activities and fraudulence. Banks also joined in the fray in search of reduction of capital requirements. The complex multi layered derivatives ensured that the same risky assets may be a part of myriad instruments and structures leading to systemic risk and cascading effect. The rating agencies relying on past data did not use models that reflected the true risk of the underlying assets in terms of the probability of default, recovery rates and default dependence. They were lax in recognizing the ascending risk in the sub prime sector. The business of rating agencies depended on the volume of transactions generated by their client, who were the originators of the engineered products. The volume was positively related to upward ratings. This in itself constituted a classic problem of conflict of interests. Performance incentives in financial institutes were designed to promote short run profitability. Lack of transparency also added to the layer of problems already mounting. In sum, the crisis may be viewed as an end result of interaction of systemic, economic and regulatory issues.

Francis A. Longstaff [9], does an empirical analysis of the pricing of subprime asset-backed collateralized debt obligations (CDOs) and looks into the cascading effects of the same on the other sections of the financial market. The results indicate the sub prime market witnessed "significant price discovery" during the crisis. The crisis of 2007 which was initiated in a specific section of a specific asset (housing) market and later pervaded to entire financial market of the US and major parts of the globe, provides an ideal platform for studying "contagion effect". Using data from the 'ABX' index for the sub prime markets, the author examines whether the effect cascaded across the markets. One of the most commonly used definitions of contagion in the literature is "significant temporary increase in cross-market linkages after a major distress event". With this definition as the theoretical support, he uses a Vector Auto Regression (VAR) framework to distinguish the pre crisis correlation between the ABX market and other markets from the corresponding figure during post crisis phrase.

The results indicate that the cross-market linkages have increased significantly post crisis. Before the subprime crisis the information contained in ABX market did not have significant explanatory or predictive power in explaining returns from other markets. In the post-crisis period, however, the ABX indices showed significant predictive power for treasury bond yields, corporate yield spreads, stock market returns and changes in the VIX volatility index. As noted by the author, the ABX indices, generally speaking, show the ability to forecast treasury yields, corporate yield spreads, stock market returns and changes in the VIX up to three weeks ahead, with strikingly high R^2 values. Such results are strong evidence in favour of a "contagion effect" having spread across markets after the crisis.

One of the major contributions of this work is that with these results it is possible to distinguish between the various existing models in the literature on contagion. One could clearly see from the length of the forecast horizon (which often ran to as long as three weeks), the view that contagion is spread via the correlated information channel, may not hold much water. The author argues that if that was the case then the price discovery would occur much more rapidly in liquid stock, bond and similar markets because of the faster spread of information. On the other hand, the fact that the results showed the ability of the ABX market to predict the trading pattern for both the liquid stock and bond markets and the market for engineered products, reinforces the model based on spread of contagion via a liquidity/financing channel.

Apart from the analytical and investigative works on the crisis, several researchers have tried to model the bubble and the market failure as well. Glaeser et al. [10] construct a simple model of housing bubbles that predicts that “places with more elastic housing supply have fewer and shorter bubbles, with smaller price increases”. The objective of their work is to find out how much of the housing asset bubble could be predicted by a rational model and related to the fundamentals, and if and when the irrational exuberance does play a role. For this purpose they construct a simple continuous time model with an underlying assumption that housing asset prices are formed by demand supply interaction, dynamically. They introduce scarcity of resources in production resulting in linear monotonic increase of prices with production. Using this model, they show that if housing supply is elastic, with a finite number of potential buyers, there is no equilibrium with the number of houses being offered exceeding the number of buyers available. Thus a rational bubble can exist only with inelastic supply.

In the next phase they turn their attention to the “irrational bubbles”. They model the same as temporary spike in the buyers’ expected price of the housing assets ascribing the same to “irrational exuberance” as described by Shiller [1]. This rise in expectation is a purely exogenous factor with a fixed life, the buyers having no knowledge of the influence of the same. The authors then propose that during bubble the exogenous factor has a multiplying effect in increasing the prices while after the factor the effect is inversed and results in multiplying the decline in prices. They also show that the interaction between the exogenous bubble factor and the supply inelasticity is similar to that of supply inelasticity and shifts in demand. The bubbles persist more in case of inelastic supply whereas in case of elastic supply they pop up much faster. The authors then proceed to analyse the data on housing prices, construction and supply elasticity during the periods of price boom and bust. Empirical analysis supports the propositions of their model.

One important aspect of any speculative bubble like the one that might lead to the housing asset crisis is the behavioural dynamics of the economic agents participating in the market. Earl et al. [11], inspect this aspect by bringing in the concept of decision cascade (as against the information cascade) and later on combine the same with Minsky's financial fragility analysis, and evolutionary economics to provide a theoretical platform for analysing the behavioural dynamics of such bubbles . The authors introduce the concept of decision cascades clearly distinguishing the same from information cascade as highlighted by Shiller [12]. The authors argue that the information cascade theory which hangs on the non-availability of information may not hold water in terms of explaining the stock and financial market phenomena as the efficient market hypothesis seems to be a reasonable approximation in light of today's media presence. The inefficiency may creep more out of the interpretation of the information than the availability of information, an issue that is not captured in the information cascade idea. This is exactly what brings in the idea of decision cascades. Decision cascade refer to the interaction effect of the decision rules as they change during the mutual transfer of information among the agents. The cascade essentially means the probability of domination of a set of decision rules over others because of social interaction norms. Any new information is thus processed according to the dominant rule set. In a speculative market such biases will ultimately lead to herding and synchronisation.

Next, the concept of the rule degradation is brought in. As decision rules cascade from agent to agent there is a possibility of "Chinese whisper" effect on the same. In effect, it means that the decision rules that ultimately prevail in sections of the market may actually be a high order derivative of the original set. Literature has shown that degradation of decision rules will happen from 'opportunism' and "tacit knowledge". Because of failure of strategies based on some decision rules, there will exist always a need for new rules in the market. People will be attracted towards newer set of strategies that are producing wealth and there will be herding towards such decision rules. Because of the way our societies have evolved, decision cascades have a great impact on our decisions, especially the speculative ones. There has indeed been numerous studies which establish the social decision making process as a cascading one.

The authors then suggest the use of agent based modelling in an evolutionary economic framework to simulate this decision cascade process. They arrive at the idea that "the degeneration of decision rules is easily modelled in a multi-agent replicator setting through imposing some kind of entropy condition on the adoption process and with replicator dynamic pay-offs of rules in relation to the population of others playing the same strategy." The evolutionary framework is an extension of Minsky [13]. The paper ultimately shows that successful modelling of decision cascade using an evolutionary framework for rule degradation can help us identify and analyse the process of speculative bubbles in a robust manner.

2.1 Agent Based Modelling in Financial Markets

We now shift our attention to some literature on agent Based literature in general to understand the sate of the art. We particularly focus on few works that concentrate on financial markets and speculative behaviour. Roszczynska et al. [14] has presented a technique based on agent based simulation. This technique is able to generate a robust measure of detachment of trading choices created by feedback, and forecast the timing of speculative bubbles in experiments with human subjects. Their work is a combination of laboratory experiments with human subjects and agent based simulations. Such a unique framework helps reveal the behavioural aspects which are crucial to formation of bubbles and whose identification ultimately may lead to preventive measures. For the experiment they used the Minority Game [15] to study the price discovery mechanism in financial markets. While already a body of literature exists on agent based simulations which model the Minority Game, the researchers shed new light on the same by incorporating experiments with human subjects. Repeating the experiment with various initial conditions, they studied the process of synchronisation into a bubble state and observed the trading strategies of the investors the interaction of which resulted in such synchronisation. To explain the process of synchronisation they take help of the "decoupling process" as explained in [16]. Decoupling essentially means trading strategies that are independent of trends. In other words the strategy is not coupled with the price movement at a particular moment of time. They used this knowledge to investigate whether decoupling plays a role in Minority game subjects and they found out that decoupling indeed played a role in synchronising the prices to reach a bubble state. They also used the additional test of false feedback post bubble stage and found that the results corroborate. Using the subject behaviour as the cue, they perform redesigned simulations on the computer incorporating the decoupling moments in their design. The results are very encouraging with almost 87% success rate in predicting the bubble states. The results are also invariant with size which indicates that the model is robust and is capable of capturing the complexities associated with higher size. This work also highlights the importance of factoring the decision biases in the agent based simulation model to be able to capture the market dynamics properly.

Rabertoa et al. [17] simulate an agent-based artificial financial market which they call Genoa market, where trading is done on one single asset by heterogeneous traders. The price discovery happens through a trading mechanism which mimics the real trading rules reasonably well. The objective of their work is to represent the trading complexities as much as possible while focusing on the finiteness of available resources. The programme allows for the pricing to be determined by demand supply interaction. Initial conditions allow the agents to start with a finite amount of cash and a portfolio of finite investment opportunities. The process does not allow creating money, there is a law of conservation of total cash in operation. In each subsequent epoch stochastic buy and sell orders of the agents are simulated. The decisions are bounded by the resource constraints and are dependent on prior period volatility also generating clustering effect. The uniqueness of the model lies in its ability to reproduce the fat tails in the probability distribution of the log returns of the assets as well as the phenomenon of volatility clustering.

Using the state of the art programming and object oriented technology, the Genoa market may be used as a platform to perform various degrees of experiment to address both research problems as well as practical issues. The authors do agree, however, that the model suffers from some lacunae which need to be addressed to increase its usability. To start with, the volatility clustering is sensitive to the size of the market. As the number of traders increases, the clustering gives way to pure stochastic volatility. Next, the model fails to capture all the stylized facts that have been empirically established about financial market behaviour. It is a well established fact from several empirical works that volatility exhibits power law decay in financial time series. The model, however, results in exponential decay. It may be added, though, that the power law decay has not been perfectly modelled as yet. Even with GARCH and ARCH models exponential decay of volatility is exhibited with correlated time steps.

The sub prime crisis was studied using a Systems Dynamics model by An et al. [18], looking at the whole problem as a multi agent complex system with the crisis evolving from the interactions between the agents. Taking cue from various literature on the recent crisis they build their system as an interaction of three subsystems namely, an aggregate banking system, an aggregate housing market and an economic environment. They followed the principle underlying the system dynamics model of Jay Forrester [19] which represents the system as a stock flow diagram. The asset-liability management decisions of the banking system are affected by the housing market as well as the economic system determining the price characteristics of the assets involved. Five classes of assets are considered: cash, short-term securities, the mortgage-backed securities, the bank-owned houses and the non-mortgage backed securities. The assets flows are modelled as linear equations taking into consideration the interest rates and asset returns associated with each class. The purchasing power of investors/homeowners which is the prime mover of the housing asset market is in turn governed by the lending policy and capacity of the banking system. This determines the stock flow of the housing market. Three kinds of housing assets are considered: houses that are currently occupied, houses owned by banks and houses not owned by banks but available for purchase. The flow model is built consistent with the banking system model taking into account the unemployment rate, mortgage interest rate, subprime loan availability and average family lifespan. The housing price is a function of the ratio of supply over demand. The economic subsystem is a dynamics between business credit and banking system liquidity. The model parameters include gross production output per unit time-period, household income per unit time-period and unemployment per unit time-period. The aggregate demand per period is a function of consumption and business environment.

The model is then simulated with given initial conditions followed by shocks thereafter. The system is perturbed by increasing the availability of subprime loan. The system observations reveal that as a result of the shock the building rates increase leading to higher expected mortgage payment per period, ultimately reaching a stage when overall mortgage payments due overshoot the affordability causing defaults to start. The model is then subject to economic stimulus in the form of government aids or stimulus money that adds to the aggregate demand. The mortgage payments return to stability provided that the stimulus is above a threshold. This in turn means that the government deficit shoots up. Overall, this model is a simple interactive system which can be used to investigate some key aspects of the crisis.

The utility of agent based models in analysing a systemic failure is brought out very clearly by Thurner [20]. Economic crisis, the author argues is a systemic phenomenon, involving complex interaction of institutions, markets, businesses, individuals and the state. The global economic structure of today lend further complexity to the system by bringing in interaction of multiple sovereign states. The complexities associated with such interactions are not possible to capture using the standard steady state economic models which thrive on general equilibrium. Hence the need for a model, which is adaptable, allows for correlation of parameters and shocks of states and does not need general equilibrium conditions. Indeed, agent based models fulfill all these criteria. The author describes in detail the process of building a model for the financial system. One main advantage is the use of 'non-representative' agents which allow the model to have varied levels of tolerance with respect to the decision making parameters. The differences between the agents can lead to results that have "macro effects". In the model there are three classes of investors: investors who allocate funds based on analysis of given information, investors who are not informed but intuitively place orders randomly and investors who place their funds in the custody of financial institutions. All these different types of investors place buy and sell orders in a market dealing with a single non-dividend paying asset. The other agents involved are the banks and regulators which govern the liquidity and leverage available in the market. The interactions between all these agents are observed as the initial conditions vary and the parameter values are allowed to evolve dynamically.

The simulation reveals that there are two scenarios which may lead to crash under leverage pressure. Firstly, the random shocks in demand caused by the uninformed investor can pull down the price of the asset much below its intrinsic value. Secondly, investment funds can take excessively large positions in the market causing concentration of risk. Both these factors may combine to form a major crisis. One of the major findings of the simulations was that in absence of regulation, both banks and investors find it attractive to increase leverage. Regulations have strange effect of producing separating and pooling equilibria under different leverage conditions. During moderate leverage they seem to work well while they seem to amplify the synchronisation during high leverage. There is an agent interaction effect which disturbs the price signal. This error emanating from interaction of agents (mutual influence) is not captured in standard models. It was also observed that during times of high leverage, even value adding strategies may act counter-intuitively adding to the crisis. One more observation about leverage was that it has a stabilizing effect when the levels are moderate. Finally, repeated simulations helped to identify the onset of crashes.

Katalin Boer-Sorbán [21] takes a detailed look at how different models can be adopted in agent based simulation framework to capture the financial market dynamics by developing an agent based simulation that would capture the behavioural characteristics of markets and investors. To develop such a system Boer-Sorbán has conducted studies on the systemic issues and general behaviour of the financial markets that are in place and pinpointed the relevant aspects of the stock market which are to be considered for modelling. They fall under two major categories: organisational and bevioural. There are delineation of six different organisational attributes that may be mapped - traded instruments, orders and quotes, market participants, trading sessions, execution systems and market rules. Apart from these well-defined observable 'hard' variables, identification of 'soft' behavioural aspects of the agents (investors, brokers and market makers) are done. The key variable for the investors is identified as "the order generation mechanism". For the brokers four distinctive variables were mapped: "order selection mechanisms, order execution mechanisms, negotiation strategies, strategies to determine transaction prices". The market makers are distinguished by "order execution mechanisms, determination and timing of quotes and handling the limit order book". With these variables Boer-Sorbán builds the framework describing the real market.

With the key variables for understanding the real market established, Boer-Sorbán studies the existing literature on artificial markets. The purpose of the study was to throw light on the relative success/ failure of various available agent based simulation models to capture the various key aspects of the real market. In the process, Boer-Sorbán has generated a conceptual framework for a taxonomy of agent based simulation models that results in extension of the conceptual framework for description of stock markets with design and implementation aspects. The analysis shows that continuous trading sessions need to be studied and incorporated in a model. The survey also reveals that most models focus mainly on the investors while ignoring the brokers and often markets as well. This has led Boer-Sorbán to propose the *Absrtacte* model of trading which takes care of the real market aspects and the best of existing ABS models.

3 A Basic Economic Model

We have built up a basic economic model to quantitatively demonstrate the role of irrational perception in investment. Rationality has its appeal in economics partly because of the fact that it is rather possible to model a rational agent. On the other hand, image of a completely directionless irrational agent is blurred in our mind. To resolve this dilemma, a thumb rule is to model a rational agent and use some departures in his behaviour from the rationality. This is called bounded rational agent which is often used when we talk about limits in rationality. This is how we can quantify departure from rationality in an agent and measure the impact of his behaviour as a function of departure from rationality. Since this crisis is about investment in housing market, we require a dynamic model to address issues associated with this crisis. In our model, agents have a simple utility function given below.

$$U(\{c_t\}) = \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{1}$$

The agents earn an income of y_t at time period t. The agents can either consume the numeraire good or can invest in an asset. The total amount of spending is equal to total income. Therefore,

$$c_t + s_{t+1} = y_t + R_t \cdot s_t \tag{2}$$

where s_t is the amount of savings which is invested into an asset of housing. R_t indicates the return on an asset at time period t which is a random variable following a Gaussian process with mean μ and standard deviation σ. The shocks are autocorrelated with correlation coefficient ρ. The agent maximizes utility function (Equation 1) subject to the budget constraint (2). We also impose additional no debt constraint that $s_t \geq 0$ for all t.

This utility function is quite common in the economics literature, so we use it to illustrate a baseline scenario. We add one element of bounded rationaility in the model. Bounded rational agents compute the expectation and the standard deviation of the interest rate fluctuation based on some few periods. The supply of housing is fixed. Therefore, price of housing is proportional to demand for investment.

What is the condition for utility maximization for an agent? The first order condition or the Euler equation, as it is popularly known, is:

$$u'(c_t) = E\left[R_t \cdot u'(c_{t+1})\right] \tag{3}$$

By solving this equation, one can derive the decision of an agent, $\{c_t, s_{t+1}\}_{t=0}^{\infty}$, based on levels of his savings and the state of the economy, demonstrated through the shock in the interest rate.

$u'(\cdot)$ is chosen from the class of constant elasticity of substitution functions,

$$u(c) = \begin{cases} \frac{c^{1-\frac{1}{\eta}}-1}{1-\frac{1}{\eta}} \text{ for } \eta \neq 1 \\ \log(c) \text{ for } \eta = 1 \end{cases} \tag{4}$$

where η is the elasticity of substitution. Typically, η is great than or equal to one.

Our framework is not novel but this is used first by Aiyagari [22] to analyse savings in a heterogenous agents model. The shocks to agents are idiosyncratic in nature. The reason for this assumption is quite straightforward. The emphasis of our modelling is not for the purpose of analysing macroeconomic fluctuations but systemic fluctuations and agents expectations. Therefore, our exclusion of economy-wide shocks is rather justified.

3.1 Algorithm for Computation

We may not be able to have any closed form solution for our optimization problem. We have an alternative of computing the numerical solution. We use the following algorithm for computing our equilibrium.

1. Discretization: We discretize the shocks using the method described by Tauchen [23]. In doing so, this methods allows us to construct a transition matrix between various shocks. We also construct a discrete grid of savings around the income of agents.
2. Initial Guess: We construct an initial guess for savings choice of agents. The consumption of an agent is the minimum of value of income and savings.
3. We compute Right Hand Side of equation 3 from the prior estimate of consumption in different states of nature. We use linear interpolation to estimate this value as and when required.
4. For various values of contemporary consumption, the Left Hand Side of equation 3 assumes different values. We solve for 3 using Newton-Raphson method.
5. We update the values for the consumption grid.
6. If the difference between previous values and updated values is sufficiently small (using L^{∞} norm), we stop. Otherwise, we go back to step 3.

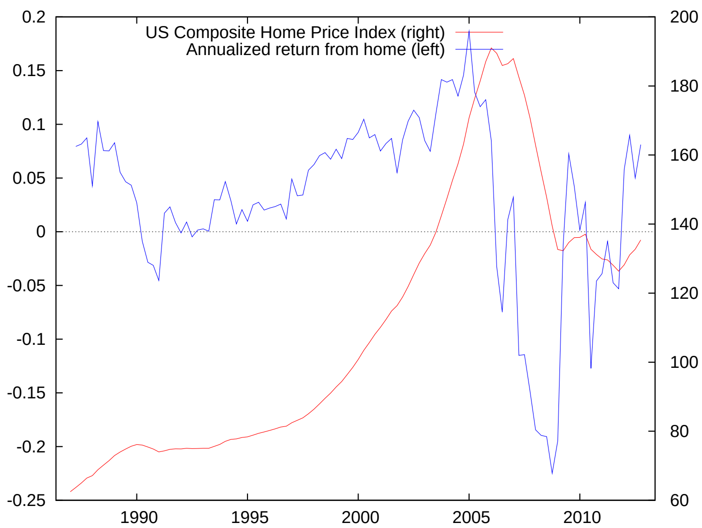

Fig. 1 National Composite Home Price Index and Annualized returns constructed (Quarterly data, U.S.)
(*For coloured figure, contact the author.*)

3.2 Calibration for Demonstrating Savings Mechanism

We experiment with some numerical values to illustrate the mechanism of our model to the readers. Risk-averseness of an agent is a monotone function of the elasticity of substitution parameter, η. We choose a conservative value of η at unity. As fas as, mean, variance and autocorrelation coefficient of shocks are concerned, we directly estimate it from the United States data. Since our focus is housing price, we look into the returns in the housing sector. National Composite Home Price Index for the United States is a series maintained by Standard & Poor. The quarterly data runs between 1987:Q1 to 2012:Q4. The mean annualized return is 3% and standard deviation is 0.08% (see fig. 1). The autocorrelation coefficient is computed as 0.67. We take discount factor as reciprocal of the mean returns.

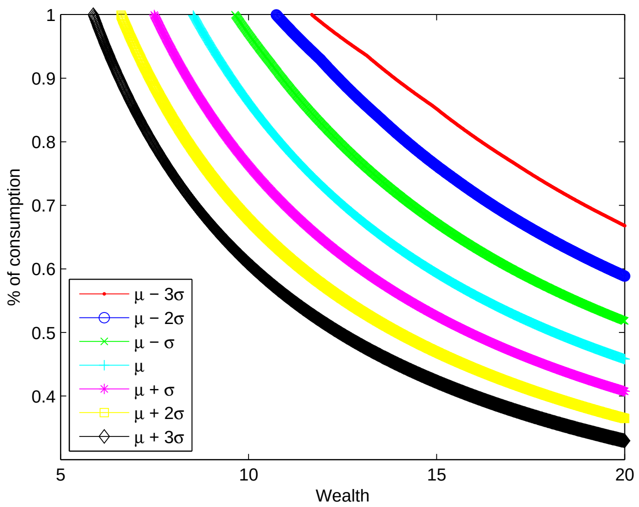

Fig. 2 Consumption as a percentage of wealth for various shocks in the return. Periodic income, without any loss of generality, is fixed at an arbitrary value of 10. The other parameters are calibrated.
(*For coloured figure, contact the author.*)

We have discretized the shock in the return into seven discrete states between $\mu - 3\sigma$ and $\mu + 3\sigma$. As autocorrelation is positive, it implies that when the return is comparatively lower, the expectation of future return is also lower. The lower expectation of future return dictates the

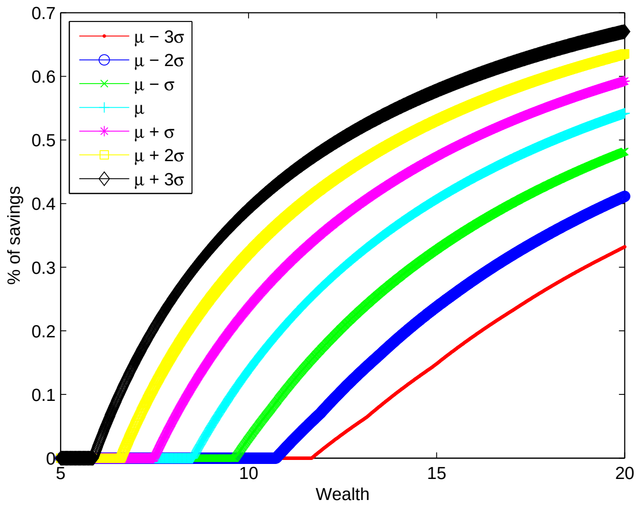

Fig. 3 Savings as a percentage of wealth for various shocks in the return. Periodic income, without any loss of generality, is fixed at an arbitrary value of 10. The other parameters are calibrated. (*For coloured figure, contact the author.*)

contemporary consumption to be comparatively higher, absolute value wise. On the other hand, an agent will have less resources available when returns are low and this motivates him to spend comparatively lower fraction of income. Overall, the contemporary consumption is dominated by the former factor compared to the latter one. Fig. 2 illustrates proportion of consumption against wealth for different values of returns shock whereas Fig. 3 portrays proportion of consumption against wealth for different values of returns shock.

3.3 Wealth Distribution and Numerical Experiments

The distribution of wealth in our model is matched with the corresponding figure of the united states. Essentially we follow an empirical approach here. Prior research [24] suggests that the wealth distribution fits the log-normal distribution at the lower tail and Pareto distribution at the upper end. The Pareto tail is restricted [25] to 10% of the population, at the most. We require to calculate mean and standard deviation of these distributions. For that purpose, we use the data [26] published by the US Census Bureau. We find that the median income of a family was approximately 60,000 USD in 2008 whereas the median family wealth was around 120,000 USD. Therefore, the median of the log-normal distribution is double the periodic income. Table 717 in [26] elaborates that approximately 10% of the population hold 1.5 million USD, which is 25 times the periodic income, in wealth. This may be one indication of the extent of power law in wealth distribution. The mean wealth is approximately 556,000 USD, which is more than 9 times the periodic income.

In our analysis, we set the periodic income to unity, without any loss of generality. The median of the log-normal distribution is set at 2 accordingly. The mean of the log-normal distribution is 9. We know that if a random variable $X \sim N(\mu, \sigma^2)$ then $\exp(X)$ follows a log-normal distribution with median $\exp(\mu)$ and mean $\exp(\mu + 0.5 \cdot \sigma^2)$. Since, we know the mean and median of the log-normal distribution, we can calculate the mean and standard deviation of the underlying normal distribution. Thereby, we derive μ as $\log(2)$ and σ as $\sqrt{2 \cdot (\log 9 - \log 2)}$. We simulate wealth of agents from this distribution ignoring the power tail for the sake of convenience.

We have enforced the idea of "irrational exuberance" through three channels: (a) increase in perceived mean, μ, (b) increase in perceived standard deviation, σ, and (c) increase in perceived autocorrelation coefficient, ρ. In the baseline case, we note the average investment of all agents when they perceive the parameters correctly. In the three numerical experiments, we implement agents' perception and note the increase in investment in each case. We increase the parameters values by 10% and note the change in investment per one percentage point change in parameters in Table 1.

4 Discussion

The results indicate that a perception of shift in mean has an explosive effect in drawing investment. A mere one percent increase in the mean returns boosts average investment tremendously by more than ten percent points. The effect of increase in mean return on agents who are at the lower range of wealth, is even more gigantic. One can safely say that a perception of change in mean will boost investment

	Mean	Median
Baseline Case	0.8010	0.0070
Increase in average return by 10%	1.6467	0.9998
	(10.6%)	(1418.3%)
Increase in standard deviation in return by 10%	0.8073	0.0094
	(0.08%)	(3.4%)
Increase in autocorrelation coefficient by 10%	0.8063	0.0105
	(0.07%)	(5.0%)

Table 1 Investment in Various Scenarios: Results from A Numerical Experiment Conducted. Percentage increase in investment from the baseline case for one percent change in the parameter value is reported in the parenthesis.

tremendously. This additional investment will definitely augment the price of house to a considerable extent. This is even more true when we consider that supply of housing is rather inelastic in the short run. This artificial boost in price will plummet down tremendously, once agents' perception falls back to the exact level causing a sharp decline in the housing prices. This is Shiller's idea of irrational exuberance. Shiller [1], for example ascribe it to "irrational exuberance" that drove the stock market bubble in the 1990s and the housing market bubble between 2000 and 2007. The speculative bubbles may be caused by "information cascades" or "decision cascades" which means that individuals in a group disregard their individually collected information because they feel that everyone else can not be wrong. He also shows how bubbles led to dangerous overextension of credit and finally to the global credit crunch. We see the same story being repeated, albeit less dramatically, when agents' perception of standard deviation in return or autocorrelation coefficient for returns changes to goad them to invest more.

We have not discussed any systemic failure in our story but provided a simple narrative when a collective perception which is different from reality may cause irrational exuberance for agents. This simple narrative may not stand the test of time if systems of financial engineering are placed to prevent and thwart any mishaps in proper time. This is where the extent of systemic flaws becomes important. A small loophole may not exacerbate a problem to a great extent but will be rectified at a higher level before becoming endemic, whereas a flawed architecture will encourage and snowball even a small problem to the level of catastrophe.

One way of looking into the dynamics of the system to analyse the nature and causes of failure is to view the entire economic process associated with the housing asset bubble as a multi-agent interaction process. In that case, the extent of failure of various parts could be measured using numerical experiment. As Farmer and Foley [27] points out that both the "econometric" as well as "dynamic stochastic models" are inadequate to map the dynamics of the crisis of this magnitude. A better alternative is the use of agent-based models. This is a computerised simulation of the decision making process of a large number of entities (agents) which may be individuals, institutions and other market participants and regulators. It is more of an evolutionary process rather than a prescriptive model. The dynamics of the complex system that led to the housing asset bubble and the subsequent economic crisis can be captured with the help of agent based simulation and multiple tiers of agents. We illustrate our case with an example.

A model can be posed in four tiers involving the buyer seller interaction in the housing market at the lowest level. A trade between a buyer and a seller takes place with the attendant instrument of price. In the second level, the buyers approach the mortgage banks for credit to purchase the house mortgaging the property. The contract between a borrowers and a lender happens at this stage. At the third level, the interaction of the large investment banks with the mortgage banks occurs. Consequently comes the creation of Special Purpose Vehicles for asset securitisation. The interaction of the investment banks with the economic system happens at the highest level. We have shown the magnitude of a bounded rational perception when there is no interaction from upper tiers. In others words, with a simple utility function the level have been modelled in this article. We have demonstrated that a small departure from reality could be a source of enormous over-investment leading to economic crisis. One may question our assumption of a wrong perception pervasive among all agents of the economy. Our results will remain essentially the same even when there are some rational and some bounded rational agents are present in the economy so long as proportional of bounded rational agents are significant in number. The magnitude of the impact of a hyped expectation has be adjusted accordingly.

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