

Aristotle's *Politics*: On Constitutions, Justice, Laws and Stability¹

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Abstract

Aristotle's *Politics* can be divided into two inquiries, each amenable to mathematical representation. The *first inquiry* assumes, probably idealistically, that individuals act in the collective interest and leads to the following theorem: polity (a rule of many *good* men) is better than aristocracy (few *good* men), and aristocracy is better than monarchy (one *good* man). The *second inquiry* assumes, more realistically, that individuals act in self-interest and leads to the following theorem as a justification for democracy: Among various systems of government, democracy (a mixed constitution with a rule of law sustained by competing factions) offers the best prospect to deliver two things at once: justice (pursuit of the common interest) and stability (obedience of the rule of law). The latter theorem implies that institutionalization of competing factions governed by good laws is likely to be just and stable. It applies to nations, corporations and towns facing the tragedy of the commons, externalities and renegeing.

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Virtue in itself is not enough; there must also be the power to translate it into action (ch. 7.3).
... When states became larger and those with arms became stronger, the number of sharers in the constitution became larger (ch. 4.13). The constitution must set the pattern for the laws (ch. 3.11). ... Law is 'a mutual guarantor of justice' (Lycophron the sophist, in ch 3.9).

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1. Introduction

Aristotle's *Politics* is not user-friendly: it is complex, pithy, abrupt, prone to digressions, and archaic. Numerous commentaries, written over centuries, seem to offer a series of partial views based on one segment, one chapter or one idea at a time. According to Kraut (2002, p. 184), some scholars even suggest that *The Politics* is "nothing more than a collection of essays on diverse political topics ... there is no organizational principle that drives the flow of ideas in the *Politics* from the beginning to end, no

destination towards which it is moving.” Barker (1978, p. 157, fn 2) states, perhaps correctly, “[w]e have to recognize, however, that Aristotle, in different contexts, says different things, which cannot be easily reconciled with one another.”

Be as it may, the essence of *The Politics* has remained obscure giving rise to an opportunity for a new approach to discover it. Fortunately, certain critical conjectures of Aristotle are so precise that they can be expressed mathematically. And once that is done, it is possible to see the unity and internal consistency of Aristotle’s ideas that connect factions with constitutions (rule of one, few or many); constitutions with justice and laws; and laws with stability and education. *The Politics* emerges from the shadows to teach us what is profound about democracy in theory, and how to construct and preserve one in practice.

Aristotle’s inquiry in *The Politics* can be divided into two parts, each amenable to mathematical representation. The *first inquiry* assumes, probably idealistically, that all men, rich or poor, are *good* meaning that they act in the common interest. A good man never knowingly sacrifices common interest for self-interest, although in specific cases he may judge wrongly due to deficient information or flawed interpretation. But given the reality of imperfect information and interpretation, who should rule? Should it be a rule of one good man or a majority? A generalization of Condorcet’s jury theorem proved in this paper tells us that a sufficiently large polity (the rule of many) would do better than aristocracy (the rule of a few), and a sufficiently large aristocracy would do better than monarchy; see Theorem 1. And that is what Aristotle asserts and that is *the* main point of the first inquiry. The inquiry continues with a bid to identify the best regime but Aristotle remains ambivalent. The jury theorem supports the ambivalence: an aristocracy of a given size (polis) may not be as good as an exceptionally qualified monarch. Thus monarchy and aristocracy, as numerical examples will illustrate, could each be better than the other. Aristotle does indeed say different things in different contexts, but for good reasons.²

² In the overview of his book, Miller (1995, p. 14) states that the *principal* task of Aristotle’s politics is to find the best or best possible constitution. Insofar as the investigation is limited to just constitutions (viz.,

Aristotle's *second inquiry* assumes, more realistically, that individuals act in self-interest. "People are much more careful of their personal possessions than of those owned communally (AP, 2.3, 1261b32, p. 108)³." The assumption of self-interest has two grave implications. First, self-interested individuals are not good men and so monarchy, aristocracy and polity would be *unstable*: when good men become self-interested, monarchy would wind up as *tyranny* (the rule of one), aristocracy as *oligarchy* (the rule of a few wealthy), and polity as *democracy* (the rule of many poor). According to Aristotle, "the good men did not remain good: they began to make money out of that which was common property of all. And to some such development we may plausibly ascribe the origin of oligarchies, since men made wealth a thing of honor (AP, 3.15, 1268b8, p. 223)." Second, tyranny, oligarchy and democracy are *unjust* in the sense that the faction in power will pursue factional rather than common interest.⁴ Thus, Aristotle's six famous constitutions go astray due to either instability or injustice. Not surprisingly, the principal theme of *The Politics* is to devise a constitution that is both just and stable.

The challenge is this: the just and stable constitution must be constructed from what exists, but what exists (*viz.*, tyranny, oligarchy or democracy) is unjust; *just constitutions* (*viz.*, *monarchy, aristocracy or polity*) do not exist when men act in self-interest. Aristotle meets the challenge in two steps. First he *constructs* a mixed constitution: a mixture of oligarchy and democracy; democracy in this paper (except in the abstract, section 2, and where noted) will carry its ancient meaning, the rule of the

monarchy, aristocracy and polity), it is only a portion of Aristotle's first inquiry. The depth of *The Politics* is to be found in the second inquiry.

³ AP shall mean Aristotle's *Politics* translated by T.A. Sinclair. X.Y refers to Book X, chapter Y of *The Politics*; e.g., 2.3 refers to Book 2 Chapter 3. The Bekker number NXn refers to page N, column X (= either a or b), line n of the Immanuel Bekker (Berlin, 1831) text; e.g., 1261b32 pertains to page 1261, column b, line 32.

⁴ Alexander Hamilton's (cited in Dahl, 1956, p. 7) observation echoes Aristotle: "Give all power to the many, they will oppress the few. Give all power to the few, they will oppress the many."

poor faction. The mixed constitution is just by construction: it caters to both the rich and poor. Second, Aristotle seeks to make the mixed constitution stable by augmenting it with just *laws*. The underlying story could be that the wealthy and poor, caught in a civil war, seek a middle ground (justice) but are apprehensive about its stability. So they decide to share in the constitution (justice) and find suitable laws that are implemented and preserved under the watchful eyes of the antagonists (stability). But with the lethal threat to the laws emanating from the rulers in high positions, the antagonists must choose laws that make violations *observable* and punishable. Such laws would include rules and procedures governing elections, the rights and freedoms, the limits on governmental authority, and checks and balances. Individuals still pursue self-interest but subject to the laws (making the constitution just), and opposing factions guard the laws (making the constitution stable).

Just laws and factional conflict are the preconditions for Aristotle's mixed constitution to materialize. Just laws offer justice (common interest), and when obeyed, stability. And factional conflict is the force that can induce obedience of the laws. So assume that history delivers contending factions willing to support just laws for their collective advantage, and there exist just laws which, if perfectly enforced, would deliver justice. Under these assumptions the question that remains is: would the laws be perfectly enforced? Experience reveals that *rulers* deviate from the laws either because they are unsure about what is lawful (poor information) or because they want to cheat (poor intent). Laws are seldom perfectly enforced. Imperfect enforcement is the only reasonable assumption to make.

A lawgiver knows or should know that imperfect enforcement of laws is the foremost problem requiring attention. Indeed, the principal problem is transgression by the rulers themselves. So what can the lawgiver propose to make the mixed constitution work? With an *intuitive* understanding of the theorems of this paper, a wise lawgiver may propose (a) installation of multiple institutions (probably the legislature, magistracy and law courts) with enough members (b) some sort of unanimity rule across institutions, and (c) the use of majority rule within each institution. Parts (a) and (b) are proposed on the basis that one institution is more likely to disobey laws than two or more institutions together would. Part

(c) is proposed on the basis, perhaps of experience, that the use of majority rule in a large body would reduce the prospect of unlawful action.

Taking (a) - (c) as the starting point, the lawgiver may inquire: for the mixed constitution to be stable, what is the maximum allowable individual-level disobedience of the laws? In other words, what is the *threshold* such that imperfectly enforced laws would deliver a just and stable constitution? The answer, summarized below, is surprising in two respects. First, the threshold is a number, and second, it seems easy to satisfy in the presence of sufficient inter-faction competition. Thus, mixed constitution is sustainable in a politically competitive society, equipped with institutions and rules described in (a)-(c), despite *considerable* imperfection in the enforcement of the laws.

Assume that (i) the preconditions (the existence of just laws and factional conflict) for a mixed constitution to be feasible are met, (ii) institutions and rules consistent with (a) – (c) outlined above exist, and (iii) the enforcement of the laws is imperfect (due to poor information, poor intent, the lack of resources to punish, etc.) Yet there is enough enforcement that potentially corrupt officials, who unlawfully place self-interest above the common interest, are deterred by *the fear of sanctions*. In particular, assume that potentially corrupt officials, deciding by majority rule, act lawfully with an *average* probability greater than 0.5; thus, some officials can be totally unlawful. Then a majority would be lawful with an even greater probability,⁵ and *a majority of a large enough body of public officials would be near-perfectly lawful*; see Theorem 2. The implication of Theorem 2 is that the mixed constitution would be near-perfectly stable. As promised, the threshold is a number (it is 0.5) and it allows considerable imperfection in the enforcement of the laws. It is the threshold because it is *least* demanding: if the average probability were less than 0.5, the opposite of what is said about the mixed constitution would happen. The deep result of Aristotle's *Politics* is this: Among various systems of

⁵ For a jury of size n , the average probability must exceed $0.5 + 1/2n$; Boland (1989). But this paper will be concerned mostly with large bodies so that the term $1/2n$ can be ignored.

government, a mixed constitution, with no dominant faction, offers the best prospect to deliver two things at once: *justice* (pursuit of the common interest) *and stability* (obedience of the laws).

There can be many mixed constitutions depending on how much the constitution leans toward oligarchy or democracy. The choice of a specific mixed constitution will depend on the context and history. Although this paper makes no attempt to find the optimal mixed constitution, it proves the following conjecture due to Aristotle: a mixed constitution that leans towards democracy (the masses) would be more stable than a mixed constitution that leans towards oligarchy (the wealthy); Theorem 3.

Aristotle continues his second inquiry because mixed constitutions, backed only by the rich and poor factions, were often unstable in reality. Factions tended to exploit their temporary military or political advantage to undermine the mixed constitution; see Examples 1 and 2 in the next section. Following Euripides, Aristotle argues that a sufficiently large middle class would help make a mixed constitution durable because its moderate policies will deter mutiny. Neither the rich nor the poor would find it worthwhile to change the constitution, or in Hardin's (2003) words, re-coordinate to a new constitution. This paper shows with the help of Theorems 2 and 3, how the middle class stabilizes a mixed constitution, or more generally, greater income equality helps stabilize the constitution.

To extract the main idea presented above and Aristotle's contribution to it, imagine there is a wise and virtuous king who can offer justice and stability during his lifetime. That may seem quite fortunate but the dependence on the wise king would probably seem as risky as the dependence for one's bread on the charity of a baker. Would the baker's off-spring be as charitable? It would be safer indeed to depend on a baker who sells bread for profit! We might ask: would the economic logic, underlying a stable supply of bread motivated by profit, extend to the political domain? Could the stability of a just constitution be anchored in self-interest rather than virtue? But given that self-interest rendered monarchy, aristocracy and polity untenable, it would perhaps be necessary to finesse self-interest without forgoing its advantages. A refined question would be this: do there exist circumstances conducive to erect an auxiliary device that is supported by self-interest but in turn controls self-interest enough to

impart a just constitution stability? Aristotle's second inquiry, developed in section 5, says yes. The opportunity arises when the factions in conflict agree to share in a constitution and erect just laws such that the fear of sanctions induces lawfulness, and thus, the stability of the constitution.

The principle of building a just constitution on the basis of conflicting self-interests was employed by Kleisthenes who in 508/7 BCE devised laws that established the famed Athenian constitution with *multiple* factions as sovereign (Aristotle 1984, Ladha, 2008); see Example 1 in section 2. More than a century later, Aristotle (born in 384 BCE) offered a comprehensive theory that connects various constitutions and of which the Kleisthenic system, being a mixed constitution, is the crown jewel. At the heart of Aristotle's work is the idea of deploying conflicting self-interests to sustain laws to control self-interest to produce a self-adjusting system. Indeed, the passage of time has not blunted the principle of building a civilization on the basis of laws that rest on conflict of interests. It is the principle that Machiavelli (1.3-4, p. 113, 1983) endorses: "in every republic there are two different dispositions, that of the populace and that of the upper class and that all legislation favourable to liberty is brought about by the clash between them." And it is the principle on which the constitution of the United States is founded. James Madison states the principle with precision: "Ambition must be made to counteract ambition (Federalist # 51 in Hamilton, Madison and Jay, 1961)."

To recap the main results of this paper, focus on the average probability of individual lawfulness. The technical results of this paper allow the use of the average probability as opposed to a fixed probability that Condorcet used. As explained later, the advantages average probability offers are the following: (i) it is realistic, more general and better (better because diversity in probabilities improves majority performance; shown later), and (ii) it makes the jury theorem compatible with game theory as probabilities do indeed vary over individuals at equilibrium. Most important, much of Aristotle's *Politics* can be tied to the average probability and the theorems that are based on the average probability. That said the main results of this paper pertaining to Aristotle's first and second inquiries, are contained in

Theorems 1 to 3. All theorems apply when rulers act independently, the number of rulers is sufficiently large (so that the Central Limit Theorem applies) and the average probability of individual lawfulness exceeds 0.5; as explained later, the assumption of independence can be relaxed.

Under the conditions stated above, Theorem 1 states that, with the rulers making errors of judgment but not intent, a polity is better than an aristocracy, and an aristocracy is better than a monarchy. Theorem 2 states that, with the rulers making errors of judgment *and* intent, a mixed constitution with its accompanying laws would offer both justice and stability. Theorem 3 states that a mixed constitution that leans towards the masses (democracy) is likely to be more stable than a mixed constitution that leans towards the wealthy (oligarchy). Theorems 2 and 3 together demonstrate that a sizeable middle class will enhance the stability of the mixed constitution. The results imply that once history delivers powerful but opposing factions, institutionalization of such factions by a mixed constitution and good laws, would yield justice and stability. It applies to nations, corporations, metropolis and villages facing the tragedy of the commons, externalities and renegeing. It probably applies with even greater force in a world that requires unprecedented levels of international cooperation in the areas of environment, human rights, trade and security.

This paper covers a substantial part of *The Politics*, but not all; see Table 1 for an overview of sorts of *The Politics*. Omitted is Aristotle's examination of different types of democracy, oligarchy and tyranny; it will have to be taken up in a different paper with a different model. Section 2 connects *The Politics* with some of the modern literature. Section 3 presents a set of definitions. Section 4 presents *Aristotle's first inquiry*. Section 5 presents *Aristotle's second inquiry* with rich and poor factions. Section 6 introduces the middle class. Conclusions follow.

2. Related literature

Despite its broad and consistent coverage of ideas, Aristotle's *Politics* has been misunderstood by well-known scholars. Even Robert Dahl, probably the best known democratic theorist of his generation, is not

accurate in his assessment of Aristotle's *Politics*. In "Democracy and Its Critics," Dahl explores the differences between ancient and modern democracies and tells us about the evolution of ideas leading up to his well-known polyarchy, which is a form of mixed constitution. In this paper, I will focus on an error Dahl makes in Chapter 2 and that is carried through the book. Dahl (1989) states that the task of the republicans is to compose a "mixed government" that "reflects and somehow balances the interests of the one, the few and the many." Dahl then states that "the more aristocratic or conservative strain of republicanism is to be found in Aristotle," and proceeds to contrast it with the modern strain that he calls democratic republicanism. What is the main difference between the two strains? According to Dahl (1989, ch 2), the aristocratic republicanism holds that the many "are more to be feared than trusted" and so the main task is to "restrain the impulses of the many." As per the democratic republicanism, however, "the element most to be feared is not the many but the few."

Based on Dahl's quoted statements and the context of his statements (Dahl, 1989, pp. 24-26), it seems reasonable to say that Aristotle would favor aristocratic over democratic republicanism and therefore would offer more power to the wealthy so as to "restrain the impulses of the many." Now, Aristotle defines what I have called in this paper mixed aristocracy (a mixture of oligopoly and democracy that *leans towards the wealthy*), and mixed polity (another mixture of oligopoly and democracy that *leans towards the poor*). Evidently, mixed aristocracy gives more power to the wealthy than poor; mixed polity does the opposite. So if Dahl were correct, Aristotle would have to pick mixed aristocracy (that leans towards the wealthy) over mixed polity. But both Aristotle and Theorem 3 contradict Dahl. Here is why. In *The Politics*, Aristotle seeks justice and stability. Both mixed aristocracy and mixed polity offer justice (both serve the wealthy and poor), but mixed polity is more stable. That is what Aristotle states and Theorem 3 proves; Aristotle's own statement appears immediately after Theorem 3 in section 5. The fact is Aristotle's second inquiry trusts neither the rich nor the poor. Instead, he proposes laws to keep the opposing factions in check. Indeed, Aristotle's second inquiry is about the violation of the laws by the *rulers*, whether rich or poor. It follows that Aristotle not

only conceptualized democratic republicanism (the one leaning towards the masses) as a mixed constitution (therefore, just) but also conjectured that it would be more stable than aristocratic republicanism (the one leaning towards the wealthy).

The above example illustrates the significance of the misperceptions of Aristotle's *Politics*. These misperceptions have grown and mutated over several thousand years limiting the use of *The Politics*. But once understood, *The Politics* can enlighten us on current issues. In the rest of this section, I will try to link *The Politics* to a handful of established lines of research in social sciences. First, *The Politics* is the forerunner to modern scholarship on political, constitutional, and legal philosophy (as a partial list, see Machiavelli 2003, Locke 1980, Montesquieu 2002, Rousseau 1988, Madison 1961, and Mill 1978 as well as Schumpeter 1950, Ostrom 1971, Riker 1987, Dahl 1989, Saxonhouse 1992, Mueller 1996, Beer 1993, Olson 1993, Ordeshook 1997, Hardin 2003, Schofield, 2006). I will take up a comparative study of Aristotle with these scholars in a separate paper. Second, *The Politics* offers mixed constitution for the betterment of rational agents. Thus, *The Politics* represents the first economic work concerned with rationality (Buchanan and Tullock, 1965); credible commitment (North and Weingast 1989; Miller 1989; Acemoglu and Robinson, 2006); peaceful (lawful) resolution of conflict under the threat of sanctions (Fearon 1995, Powell 1999); and stability. The role of the middle class or that of income equality in the stability of the mixed constitution has continued to be a topic of current interest (Moore 1966, Przeworski et al 2000, Boix 2003, Acemoglu and Robinson 2006).

Third, scholars of social choice theory (Arrow 1963, Sen 1970, 1977a, Gibbard 1973, Satterthwaite 1975, McKelvey 1976, Schofield 1978, Riker 1982) have suggested, with reference to majority cycles, that unless preferences are unidimensional or aligned in a nice way (Plott, 1967) “anything can happen in a democracy”. The conclusion gave rise to a need for a theoretical defense of democracy. Aristotle's *Politics* offers a justification for democracy: a mixed constitution offers the best prospect of both justice and stability; Theorems 2 and 3. An implication of Aristotle's *Politics* is that unlawful alternatives are forbidden: majority cycles cannot include unlawful alternatives.

3. *Background and Definitions*

Aristotle's inquiry builds upon *his* definitions of faction, state, citizen, constitution, a good man, justice and laws. A reader may not fully agree with a definition or two, but would probably do well to hold fire and grant Aristotle the liberty to define terms at pleasure. The reader is however free to judge whether the results obtained from these definitions are in accord with Aristotle's reputation.

To motivate Aristotle's definitions, suppose the rich and poor, being in conflict, are both armed, or one is armed and the other relatively unarmed. At the end of the conflict, a constitution is installed that may favor one group more than the other depending on their relative powers. The constitution may be thought of as a bargaining agreement reached under the threat of war. Of course, if one side is unarmed, the threat would be hollow giving the other side the opportunity to write the constitution as it pleases. The definitions follow.

Faction. Aristotle's unit of analysis is a faction. Members of a given faction (e.g., the rich) have common interest. *Members of different factions have both common and opposing interests.* Like players in a Prisoner's dilemma game, a coordination game, or a bargaining game, different factions (e.g., the rich, poor, middle),⁶ though in conflict, have the opportunity to gain from cooperation or peaceful resolution of a potential conflict.

A *state* consists of people, citizens and non-citizens, who share in the resources and territory of the state and who help make the state economically and defensibly sustainable. A *citizen* is one who is "entitled to participate in office, deliberative⁷ or judicial" of the state (AP, 3.1, 1275b13, p. 171). "What

⁶ "Inequality is everywhere at the bottom of faction, for in general faction arises from men's striving for what is equal (AP, 5.1, p. 298)."

⁷ By a deliberative office Aristotle probably meant both legislative and executive. An executive position is deliberative because it applies laws to specific cases. Aristotle's consistent distinction between deliberative and judicial offices is probably because a deliberative office is often limited (term limit)

effectively distinguishes the citizen proper from all others is his participation in giving judgment and in holding office (AP, 3.1, 1275a22, p. 169).” The citizens *taken together* are the *sovereign*. The *rulers*, at a given time, are the various officers of the state drawn from the citizen-body, and the *ruled* are the members of the state who are not rulers. The sovereign is the supreme ruler above all officers and laws.

A *constitution* is a collection of factions with an agreement for (a) “the determination of sovereignty,” (b) “the distribution of offices,” and (c) the pronouncement of the ends that the factions wish to realize.⁸

The factions which benefit from the distribution of offices are said to have a share in the constitution. To illustrate the definition, suppose there are two factions R (Rich) and P (Poor). These factions would yield three constitutions: {R}, {P} and {R, P}; of course, each has to be supplemented with the allocation of offices and the aim of the constitution. For Aristotle’s *Politics*, a constitution is a collection of one or more factions that share power, whereas laws specify the principles and processes of governance. Thus, Aristotle’s laws are akin to modern constitutions. The following three examples, covering a mixed constitution, an extreme democracy and a rule of one, will be useful in understanding Aristotle’s *Politics*.

Example 1. Mixed Constitution. The Kleisthenic constitution of 508/7 BCE, commonly known as the Athenian Constitution, represents a bargain between the warring nobles and the people (Aristotle 1984, Ladha, 2008). It (a) defined the rules of citizenship broadly; the citizen-body being the

whereas a judicial position is unlimited (no term limit), a citizen can be a juror without limit; see footnote 5, p. 170.

⁸ Aristotle defines constitution in several places in *The Politics*. The most complete one is the following (AP, 4.1, 1289a11, p. 237): “A constitution is the arrangement which a state adopts for the distribution of offices, and for the determination of sovereignty in the constitution and of the end which the particular association aims at realizing.” The word association refers to the state: “the state is a kind of association – an association of citizens in a constitution (AP 3.3, 1276a34, p. 176)”

sovereign, (b) defined the rules for the distribution of offices (the poor dominated the Assembly, the rich got the Supreme Court, and all shared in an elected Council of 500), and (c) sought to advance the interests of the competing factions through justice,⁹ liberty,¹⁰ peace, trade and defense. The Kleisthenic constitution was a mixed constitution that *institutionally* empowered all factions and lasted for nearly thirty years during which Athens won the Battle of Marathon in 490 BCE and the Battle of Salamis in 480 BCE, both against Persia (Hansen, 1991). The mixed constitution was transformed to an extreme democracy, but was reinstated in 400 BCE and lasted for another eighty years.

Example 2. Extreme Democracy. Two consequences of the naval victory at Salamis are noteworthy: the rise of the Athenian Empire and transformation of the domestic politics. The Navy employed nearly 50% of the Athenian citizens to operate 200 triremes; the peak population of Athens was only 60,000 male citizens, and each trireme had to be manned by 200 sailors. Thus the navy inadvertently organized the poor and the poor became the dominant faction. Around 460 BCE, an extreme democracy was installed representing a substantial change in the constitution: (a) the citizenships rules were changed (now both parents had to be citizens), (b) the distribution of offices was changed: the nobles were stripped off much of their political power,¹¹ and (c) the state sought to advance the interests

⁹ Aeschylus, a contemporary of Kleisthenes, celebrates the establishment of justice in The Oresteia. In the end, a majority of the Areopagus, the Supreme Court of ancient Athens, frees Orestes, marking the installation of justice that is in the *common interest* of the parties caught in a never-ending blood feud.

¹⁰ In 1644, English poet John Milton published *Areopagatica* calling for freedom. Milton prods the English nobility to protect freedom as the Areopagus, a stronghold of the Athenian nobles, did.

¹¹ Around 460, the Areopagus was reduced from being the Supreme Court to a mere homicide court; see Fine (1983) and Hansen (1991).

of the dominant faction: the poor.¹² Plato grew up in an extreme democracy that in the end executed Socrates after convicting him of polluting the minds of the noble youth responsible for the coup of 404. After the Peloponnesian War (431 to 404 BCE) against Sparta, which Athens lost, the Athenians agreed to return to a mixed constitution, empowering the rich, middle and poor, in 400 BCE (Fine 1983, Hansen 1991, Kagan 1990). Aristotle probably wrote *The Politics* around 350 BCE. The Macedonian juggernaut of Alexander the Great crushed the mixed constitution of Athens in 322/1 BCE.

Example 3. Rule of one. The constitution of Alexander the Great had one citizen-sovereign (Alexander), it distributed offices as Alexander pleased (lacking citizenship, no other person was *entitled* to seek any office), and it sought to dominate the world and prosper through plunder. Thus, the definition of a constitution allows a lopsided “agreement” between the powerful and the feeble.

Connections between citizenship, constitutions, sovereignty and factions

Recall, a *citizen* is entitled to participate in office and a constitution determines the distribution of offices. “A citizen, therefore, will necessarily vary according to the constitution (AP, 3.1, 1275a22, p. 170).” “[I]t often happens that one who is a citizen in a democracy is not a citizen in an oligarchy (AP, 3.1, 127b32, p. 168).” But “in every case the citizen-body of a state is sovereign (AP, 3.6, 1278b6, p. 187).” Given that a state could have one, few or many citizens, it follows that “sovereignty necessarily resides in one man, or in a few, or in the many (AP, 3.6, 1279a22, p. 189).”

It might seem that there are three basic constitutions: those with sovereignty residing in one, few and many. The impression this creates is misleading because a large enough set can have many subsets of a few and many subsets of many. To see this, suppose there is a set C of, say, 10 factions.

¹² “For the Athenian sea-power in the Persian wars was due to the Athenian people; this gave them a great opinion of themselves, and they chose inferior men as popular leaders when respectable men pursued policies not to their liking (AP, 1274a11, 2.12, p.161).”

Then the power set of C will contain $2^{10} - 1 = 1023$ subsets of C: 10 subsets with one faction each, 45 with 2 each, ..., and 1 with all 10. Thus, the set C of 10 factions permits 1023 possible constitutions. This number will greatly increase when the various ways in which each constitution can assign state's offices and goals are included.

In reality, the factions differ depending on income (rich, middle and poor), armor (armed and unarmed) and trade (agriculture, commercial and mechanical workers). Some factions may distinguish themselves on the basis of family, virtue and education. Thus there may be many constitutions in the real life depending on which factions get to share in the constitution. "The reason for the plurality of constitutions lies in the plurality of parts [factions] in every state (AP, 4.3, 1289b27, p. 241 and 4.4, 1290b38, pp. 246-7)."

Parsimony requires that the number of constitutions be reduced by defining some basic (disjoint) constitutions. It is difficult, however, to compose disjoint constitutions from factions with overlapping memberships; "the same people may be defenders, farmers, skilled workmen, and judges and deliberators (AP, 4.4, 1291a33, pp. 248-249)." Fortunately, constitutions based on wealth offer a disjoint set. In Aristotle's words, "the same people cannot be both rich and poor, and that is why the prime division of a state into parts [factions] seems to be into poor and the well-to-do. ... these two parts appear to be as opposites ... So the constitutions are accordingly constructed to reflect the predominance of one or other of these, and there seem to be two constitutions – democracy and oligarchy (AP, 4.4, 1291a33, pp. 248-249)."

What occurs is that many are poor and few are rich. So "a *democracy* exists whenever those who are free and are not well-off, being in majority, are in sovereign control of the government, an *oligarchy* when control lies with the rich and better born, these being few (AP, 4.4, 1290b7, p. 245; emphasis added)." These definitions of democracy and oligarchy are maintained throughout this paper; the definitions are consistent with the present-day left-right conceptualization of politics.

Two small steps will take us to Aristotle's famous six-way classification of constitutions. First, to get to three from two (oligarchy and democracy), add the rule of one (this person is likely to be rich!) Second, to get to six from three, let each constitution possess one of two aims: pursuit of the common interest and pursuit of the factional or self-interest; the specification of the aim is required to fulfill part (c) of the definition of a constitution. Hence, there are six *disjoint* constitutions: rule of one, few (wealthy), and many (poor); each with two possible aims.

Aristotle's six constitutions, good man and good citizen

The right constitutions, the ones that aim at the collective interest, are *defined* to be *monarchy* (the rule of one, also referred to as kingship), *aristocracy* (the rule of a few) and *polity* (the rule of many). The corresponding *deviations or wrong* constitutions are *tyranny* (the rule of one), *oligarchy* (the rule of a few) and *democracy*¹³ (the rule of many); wrong because they aim at the good of the rulers (AP, 3.7, 1279a22, pp.189-190).

Aristotle's six disjoint constitutions are a substitute for a plethora of overlapping constitutions arising from a multiplicity of factions. To examine the nature of outcomes under each constitution, Aristotle examines the role the citizens play in the various constitutions. Aristotle begins with a definition of a good citizen and good man. With the distinction between the two not being crystal clear, I will attempt to fill in a few gaps. It is this distinction between a good man and good citizen that partitions Aristotle's two inquiries.

“[T]he task of all citizens, however different they may be, is the stability of the association, that is, the constitution. Therefore the virtue of the citizen must be in relation to the constitution (AP, 3.4, 1276b20, p. 170).” *A good citizen of one of the deviant constitutions will act in the factional interest.*

¹³ Aristotle's characterization of democracy (the rule of one faction, viz., the poor) as wrong is no reflection on modern-day multi-faction “democracies.” Indeed, mixed constitutions are multi-faction democracies.

Thus, the virtue of a citizen of a democracy lies in the preservation of democracy and that means pursuit of the factional interest of the poor. Likewise, the virtue of a citizen of an oligarchy lies in the preservation of oligarchy and that means pursuit of the factional interest of the few wealthy. Clearly, the two virtues, one working for the poor and the other for the wealthy, cannot be the same and therefore neither is perfect virtue. Observe the necessity of disjoint constitutions. If two constitutions had overlapping membership, then virtue in relation to one could be consistent with that in relation to another and so either could qualify for being a perfect virtue.

A *good man* “is good because of one single virtue which is perfect virtue (AP, 3.4, 1276b20, p.179).” Clearly, perfect virtue must be invariant over constitutions. If no pair of constitutions has a common element, then perfect virtue would not exist. If two or more constitutions have a common element pertaining to virtue, then that would be a candidate for perfect virtue. The three right constitutions (monarchy, aristocracy and polity) have one common feature: each aims at the collective interest. Therefore, the perfect virtue is to pursue the collective, not factional, interest. It follows that a *good man* pursues, whether as a monarch, an aristocrat or a member of the polity, the common interest of the society.¹⁴ Therefore, a good citizen of a democracy, an oligarchy or a tyranny, seeking factional interest, cannot be a “good man” in the sense defined. A good citizen of one of the right constitutions, however, would be a good man.

The virtue of a good man manifests itself when the good man is a ruler (as opposed to being ruled) because collective interest is a matter of public policy and the rulers make public policies. Thus, “the virtue of good ruler [e.g., an ideal monarch] and good man is the same (AP, 3.4, 1277a12, p. 181).” Now Aristotle says that a good ruler is both good and endowed with practical wisdom, whereas practical

¹⁴ Lupia and McCubbins (1998, p.41), as they develop their theory of persuasion, start with Aristotle’s *Rhetoric* that invokes *good men*. “We believe good men more fully and more readily than others (Barnes, ed. 1984, cited in Lupia and McCubbins (1998)).” Aristotle seems to be making a come back through mathematical modeling.

wisdom is not essential to a citizen.¹⁵ What does Aristotle mean by practical wisdom? *I think* it has to do with finding a way to pursue collective interest in practice, that is, in the real life. A good ruler, acting as a lawgiver, will have the practical wisdom to deliver collective interest at *equilibrium*. That is, a good lawgiver will create proper incentives so that the citizen responses under these incentives will be in the common interest.

Table 2 defines Aristotle's six disjoint constitutions in the language of Abraham Lincoln's Gettysburg speech "... that the government *of* the people, *by* the people, and *for* the people, shall not vanish from this earth (emphasis added)." The table also serves as a summary of the discussions thus far.

[Table 2 about herer]

Justice

With focus on just and stable constitutions, Aristotle proposes only three types of justice for his six constitutions: democratic justice, oligarchic justice and absolute justice. That is because (a) all three right constitutions in the second column of Table 2 will conform to the absolute justice, and (b) tyrannical justice, although not meaningful, can be viewed as a special case of oligarchic justice.

All three forms of justice require the citizens to take turns at ruling (equality in ruling); of course, under monarchy, the lone citizen-monarch continues to rule. But inequality of citizens and non-citizens is also just: citizens and non-citizens do not take turn at ruling; that is so even today throughout the world. Thus, "justice is equality ... only for those that are equal. ... Inequality is also thought to be just ... only for the unequal."

Taking turns at ruling is a means to an end, and the end is also a part of justice. Democratic justice seeks the factional interests of the poor whereas oligarchic justice seeks the factional interest of the wealthy. In contrast, absolute justice seeks the common interest of both the poor and wealthy.

¹⁵ "[T]he only virtue special to a ruler is practical wisdom: all the others must be possessed, so it seems, both by rulers and by ruled (3.4, 1277b16, p.182)."

Unless otherwise stated, justice in the rest of the paper will mean absolute justice. *Justice requires (a) equality of the citizens in ruling, and (b) the pursuit of the common interest.* Considering that equality of the citizens in ruling will not be an issue, I follow Aristotle's convention that *to pursue the common interest is just (absolutely just), and to pursue factional interest is unjust.* "In the state the good aimed at is justice; and that means what is for the benefit of the whole community (AP, 3.12, 1282b14, p. 207)." A constitution is just if the sovereign pursues the common interest, and unjust if the sovereign pursues the factional interest (AP, 3.6, 1279a16, p. 189; 3.7, 1279a22, p. 189).

Laws for Aristotle's first inquiry

Laws exist to protect the constitution and promote the interests of its constituent factions. "As constitutions vary ... the laws too invariably vary ... Laws framed in accordance with one of the right types of constitution will inevitably be just, but if according to one of the deviations, unjust (AP, 3.11, 1282a41, p. 206)." Laws vary with constitutions because tyranny, oligarchy or democracy will obey neither just laws nor each other's unjust laws; they will obey only their own unjust laws. Monarchy, aristocracy and polity will obey only their respective just laws.

A deeper review of laws, relevant to Aristotle's second inquiry, will be taken up later. Presently, two observations would suffice for the first inquiry. First, the goal of *The Politics* is to discover constitutions that are just and stable. Tyranny, oligarchy and democracy, taken individually, are dead ends: each, being a one-faction constitution, is unjust. In contrast, each citizen of monarchy, aristocracy or polity, being a good man would welcome rather than transgress just laws. Just laws relevant to the first inquiry are catalogues of collected wisdom offering time-tested knowledge; laws are not required to force good men to act in the common interest. Second, "the laws enunciate only general principles and do not give day-to-day instructions on matters as they arise (AP, 3.15, 1286a7, p.221)." So officials must apply laws to specific cases.

4. Aristotle's First Inquiry

The question is: “ought the power to rule to rest with the one best man or with all (AP, 3.15, 1286a21, p.222)?” Consider a polis of n good men. Some are wealthy, some poor but all are *committed* (Sen, 1977b) to act in the common good of the polis. In particular, each citizen subscribes to the just laws of the polis. Laws, however, are statements of general principles and procedures, and must be applied to specific cases. Citizens depart from the provisions of the law only in situations the law itself does not cover. And when they depart, only their judgments, not intents, are subject to error.

In each specific case, there are two alternatives C and F. The alternative C is in the common interest and the alternative F is in the factional interest of either the wealthy or poor. The citizens while knowledgeable do not know with certainty the better alternative; thus, some citizens might erroneously judge F to be better for the society. Decisions are made by majority rule.¹⁶ The goal is to compare majority accuracy of polity with that of aristocracy, and majority accuracy of aristocracy with that of monarchy.

Aristocracy versus monarchy

Omitting a full description of game-theoretic technical details, suppose *at equilibrium*¹⁷ p_a is the probability that each of the n_a aristocrats *independently* votes for the better alternative C.¹⁸ The case of

¹⁶ Aristotle (AP, 4.8, 1294a9, p.260) states that “[t]he principle of majority decision belongs to all three: in oligarchies and in aristocracies and in democracies.” The same would apply to polity as well; the polity is not mentioned because of the context.

¹⁷ To avoid irrelevancies, the equilibrium under consideration is one at which wisdom or knowledge aggregation occurs.

unequal p_a 's and correlated votes will be considered shortly. Let P_A be the probability that a majority of the aristocrats votes for C at the above equilibrium. Finally, let p_m be the probability that the monarch votes for the better alternative C. To compare aristocracy and monarchy, we need the following.

Condorcet's Jury Theorem

Suppose at *equilibrium* each citizen votes *independently* for the better alternative with probability $p_a > .5$. Then, (a) a majority of the citizens would do better still ($P_A > p_a$), and (b) the majority will tend to become infallible with the size of the group, that is, P_A will approach 1 as n approaches infinity.^{19,20}

To illustrate, let the monarch, whose intent to act in common interest is never in doubt, be subject to errors of judgment in specific cases. Let the monarch be the wisest (or most informed) of them all. So set $p_m = 0.999$. Thus in 99.9% of the cases the monarch would be able to discover the common interest. The aristocrats also act in the common interest and are also prone to judgmental errors. Let $p_a = 2/3$ for the aristocrats at equilibrium; the monarch is not treated as a part of the aristocracy. The aristocrats are wise but not as wise as the monarch. Consider two aristocracies of sizes 51 and 101. A majority of the aristocracy of size 51 will choose the better alternative with probability 0.985 (not as good as the monarch); it is a straightforward application of the binomial distribution. But a majority of the

¹⁸ Readers familiar with the game-theoretic literature on Condorcet's jury theorem will notice that the equilibrium probability p_a fully reflects each voter's information including that extracted on the presumption of being pivotal; see Ladha, Miller and Oppenheimer (1997).

¹⁹ If $p < 0.5$, then $P_n < p$ and $P_n \rightarrow 0$ as $n \rightarrow \infty$. If $p = 0.5$, $P_n = 0.5$ for all n . The theorem holds for any n ; when n is even, we need to assume that ties are broken by tossing a fair coin. For a proof, see Ladha (1992).

²⁰ Waldron (1995) applies Condorcet's jury theorem to chapter 11 of Book 3 of Aristotle's *Politics* titled, "The wisdom of collective judgments." Waldron does not compare Aristotle's constitutions.

aristocracy of size 101 will choose the better alternative with probability .9994 (better than the monarch). The example, as noted in the introduction, makes it clear why Aristotle remains ambivalent in his inquiry of the best constitution. Indeed, the jury theorem can attest to Aristotle's conclusion that monarchy would be best when the monarch is of such "superlative virtue" that "the rest are simply not to be compared with" him (AP, 3.13, p. 213, 1284a3). For the monarch to be incomparable, it ought to be that he acts in the common interest with probability 1 because any probability less than 1 can be exceeded by a majority of a sufficiently large aristocracy (or polity). In other words, such a man must have perfect information and must never misinterpret. What does Aristotle call such a man? "We may reasonably regard such a one as a god among men ...: they are themselves law (AP, 3.13, p. 213, 1284a3)." Thus, the proposition that monarchy is the *best* form of constitution is no more than an extreme theoretical possibility; the same logic will apply to aristocracy as well. I have thus explained the following remark due to Newman (1973; cited in Miller, 1995): "The answer is that the best constitution will assume the form of an Absolute Kingship or the more equal form of an Aristocracy of *spoudaioi* [excellent persons], according to circumstances."

Returning to Condorcet's jury theorem, note that *only* part (b) of it is required to establish the superiority of aristocracy over monarchy. Part (b) implies that *a majority of a sufficiently large aristocracy would do better than monarchy*. Unlike part (a), which is true for any n , part (b) is a version of the law of large numbers.

Polity versus aristocracy

By the same logic, a majority of a sufficiently large polity would be better than a majority of the aristocracy. To see this, let the majority of the aristocracy be correct with some probability $P_A < 1$. Then, by part (b) of Condorcet's jury theorem, there exists an n_p such that for any polity of size greater than n_p , *the majority accuracy of the polity will exceed the majority accuracy of the aristocracy* P_A .

Condorcet's jury theorem assumes that each member of the polity votes for the better alternative with the same probability p_a . *What would happen if the probability of individual accuracy differed from person to person?* It will be shown that part (b) of Condorcet's jury theorem, which is the part we need, will hold with even greater force: for a sufficiently large voting body, a majority with unequal p_{ai} 's, $i = 1, \dots, n$, will do better than a majority with equal p_{ai} 's provided that average of the p_{ai} 's in both cases are the same. That is, for a sufficiently large group, it would be better if individual accuracies were not all equal. Proposition 1 will prove precisely this result.

Proposition 1. Consider a voting body such that its members vote independently. At equilibrium, suppose voter i votes for the better alternative with probability p_i for all i . Denote the probability that majority votes for the better alternative with P_A . Then for a sufficiently large voting body, $P_A(p_i$'s are not all equal) $>$ $P_A(p_i$'s are all equal). That is, diversity improves the voting performance.

Proof. See Appendix.

We have the following theorem.

Theorem 1 (Aristotle's first inquiry)

Consider a society of good (virtuous) men having to decide between a pair of alternatives by majority rule. Suppose at equilibrium each citizen votes *independently* for the better alternative with an *average* probability greater than 0.5. Then a majority of a sufficiently large polity (the rule of many) would do better than a majority of a given aristocracy (the rule of a few), and a majority of a sufficiently large aristocracy would do better than a monarchy (the rule of one).

Theorem 1 formalizes the following conjectures of Aristotle: When the "rule of the majority who are all good men [is described] as aristocracy, and the rule of one as kingship, then aristocracy in a state will be preferable to kingship (AP, 3.15, 1286a36, F223)."

Two questions may be raised about the assumptions of Theorem 1. First, must all men be good for Theorem 1 to hold? The answer is that all men need not be good. So long as the average

probability exceeds 0.5, Theorem 1 will hold.²¹ Second, what would happen if the votes were correlated? Correlation between votes can arise from common information or common models used for the interpretation of information. If the votes were pairwise perfectly correlated, such a polis would boil down to being one citizen. A monarch could do better than such a polis. Even if the votes were highly, but not perfectly, correlated, a monarch could still do better. Correlation effectively reduces the number of citizens often to a handful of “schools of thought” (Ladha, 1992). Assuming for simplicity that the members of a school are perfectly correlated but the schools are mutually independent, a group of many citizens would shrink in size to the number of independent schools. As a result the advantage of Condorcet’s jury theorem, which is based on numbers, would diminish. A polity that subscribes to a handful of independent schools of thought may do worse than an aristocracy that entertains many schools of thought. And an aristocracy that subscribes to a handful of schools of thought may do worse than an exceptionally qualified monarch. The above discussion is consistent with Aristotle’s reluctance to name the best constitution although he talks about the best constitution on several occasions.

5. Aristotle’s Second Inquiry

The aim of this section is to develop a constitution that is just and stable even as the officials err both in their judgments and intents. The task is difficult because justice demands pursuit of the common interest but individual intents flow mainly from self-interest. A great deal of microeconomics, game theory, political science and philosophy inquires into ways to promote collective interest by constraining self-interest. Familiar ways include promotion of competition; intelligent regulation of monopolies, externalities and imperfect information; and mechanism design, a branch of game theory. Ancient lawgivers, at least the successful ones, conceptually understood these approaches as they sought to control

²¹ The result is akin to Miller’s (1986) result. Miller defines majority outcome under perfect information as the “correct” one, and shows that it will be attained, with probability close to 1, under imperfect information if the average probability exceeds 0.5.

factional interests. Aristotle's *Politics* offers a theory that distills the experiences of his times and seems relevant today.

To present the heart of Aristotle's *Politics*, I begin with Aristotle's definition of laws. This is followed by the definition of stability. As can be expected, the main device to control self-interest will be laws and the main property to investigate will be stability. The investigation is carried out for just, unjust and mixed constitutions. Theorem 2 presents the conditions for stability when the laws associated with the mixed constitution are imperfectly enforced. Should the conditions, to be outlined in the Axiom of Mixed Constitution, be met, Theorem 2 would enable imperfect citizens to operate a near-perfect society. Yet to guard against the risk of a short-lived constitution, the Axiom must be fulfilled at each instant. Two related events can increase stability. First, a mixed constitution that leans towards democracy would enhance stability; Theorem 3. Second, addition of a sizeable middle class would help make the validity of the Axiom durable and the constitution stable; see section 5. In plain English, inter-faction competition and greater economic equality enhances the prospect of justice and stability. Finally, the last part of this section and section 6 seek to connect the various conjectures and ideas of *The Politics* with Theorems 2 and 3.

Laws. "Laws distinguishable from descriptions of constitutions are those according to which rulers shall rule and *shall watch out for those that transgress them* (AP, 4.1, 1289a11, p. 237, emphasis added)."²²

To bring to light the depth of Aristotle's definition of laws, observe that the definition applies to all constitutions. As per the first part, that "the laws are those by which rulers shall rule," the laws

²² The definition of laws being important, I offer two other translations of the original: "But laws are not to be confounded with the principles of the constitution: they are the rules according to which the magistrates should administer the state, and proceed against offenders (Jowett, p. 147, IV i)." Also "Laws, as distinct from the frame of constitution, are the rules by which the magistrates should exercise their powers, and should watch and check transgressors (Barker, p. 156, IV i)."

specify the rules of official appointments; the rules of transition; the administrative structure and procedures for legislative, administrative and judicial branches; the rights of the citizens; and more generally, all principles and procedures the rulers require to rule. Some of these processes and procedures are topics of discussion in ch. 14 (legislative, deliberative), ch. 15 (executive) and ch. 16 (judicial) of Book 4 of *The Politics*. To get a sense of these in the modern literature, see Ordeshook's (1997) set of rules for setting up a democracy (in the modern sense of democracy). To get a sense of how the laws, as processes and procedures, could preserve the intent of the founders of the constitution, see McCubbins, Noll and Weingast (1989). Moreover, given that just laws seek common good, it would be fair to say that if individual rights, including property rights, advance common good, then these rights would also be a part of just laws; this is a point Miller (1995) makes. What Aristotle's laws do not specify are the citizens and their aims; constitutions do that.

The second part of the definition, that laws are those by which rulers "shall watch out for those that transgress them," says that the laws help *watch out and punish* the transgressors; without the possibility of punishment *watching out* would be futile. Now the potential transgressors include the rulers and the ruled. Assume for the moment that the rulers possess enough force to deter transgression by the ruled, but the ruled lack the resources to deter rulers. So transgression by the *rulers* remains an issue. Consider Aristotle's six constitutions. Clearly, transgression by *good* rulers of just constitutions is not a problem so long as the rulers remain good; good rulers, by definition, would obey just laws. In contrast, transgression by the rulers of unjust constitutions is a problem that is not easily solved. Such rulers (a tyrant, an oligarchy or a democracy), with men rather than laws as sovereign, would frame laws to suit the ruling faction. Therefore *watching* transgression by the *rulers* of Aristotle's six constitutions is immaterial: just rulers need not be observed, and unjust rulers will make the observed lawful. The point however is that the definition of laws applies to all six constitutions; the second part of the definition of laws is redundant when the rulers are good or no force exists to check the rulers. As explained shortly, the same definition would apply to the laws associated with a mixed constitution.

Stability. A constitution is stable if the laws associated with it are obeyed.

An inquiry into the stability of a constitution is an inquiry into the obedience of its laws, and thus into the incentives the laws create as the *rulers* and *ruled* decide whether to act lawfully or not. We can imagine rulers who promise to abide by laws that benefit all, but then renege because there is no one to punish them and because it is to their advantage (monarchy, aristocracy and polity, as a result, are unstable). As North and Weingast (1989) would say in the game theoretic language: such rulers cannot make credible commitment. Stability is closely related to the idea of credible commitment: both pertain to the obedience of laws (or the agreement between factions that creates laws) by self-interested players as they consider the payoffs from obedience and disobedience.

When the rulers have the resources to constrain the ruled, tyranny, oligarchy and democracy would be stable because the ruled cannot defy, and rulers will find obedience of the laws self-serving. Deviation from the laws, if any, would arise from errors of judgment, but such errors could be contained by the use of majority rule in the case of oligarchy and democracy; the Condorcet's jury theorem applies when the rulers have a common goal. That said, it is time to consider mixed constitutions with focus on both justice and stability.

Composition of a just constitution

Where should the sovereign power of the state reside so as to obtain justice? "With the mass of the people? With the rich? ... With one man, the best of all (AP, 3.10, 1281a11, p. 200)?" Aristotle rejects each alternative. The poor would use their numerical strength to redistribute the property of the rich (which is unjust being contrary to common interest), and the rich (few) would plunder the poor (which is also unjust).

Instead, Aristotle proposes a mixed constitution that is a mixture of oligarchy and democracy. In the mixed constitution (a) the sovereign or the citizen-body consists of both the rich and poor, (b) the offices or institutions are distributed to both factions in an intelligent fashion (see footnote 25 for an illustration), and (c) the aim is the common good. The mixed constitution is *just* because of (c): “[t]he aim of the mixture is merely to have regard to the interests of both well-to-do and poor (AP, 4.8, 1294a9, p. 260).” According to Aristotle, “[a] constitution which is really well-made combination of oligarchy and democracy ought to look like both and like neither (AP, 4.9, 1294b14, p. 2630).”

The laws associated with the mixed constitution are those by which the rulers (viz., the rich and poor factions) shall rule (conduct the affairs of the society) and punish any transgressor. Obviously, when a faction itself is a transgressor, the other faction(s) or the institutions of the state (e.g., the judiciary) must punish. Moreover, the punishment (including the threat of violence) should be sufficiently large that it deters transgression: the factions should obey laws at equilibrium. In the words of Aristotle, a mixed constitution “should be kept stable by means of itself and not through outside agencies (AP, 4.9, 1294b36, p. 263).” It would be doing that “when no section whatever of the state would even wish to have a different constitution (p. 263).”²³ Figure 1 should help clarify the point.

Suppose the just laws of a mixed constitution are *perfectly* enforced by players F1 and F2 themselves. F1 is wealthy and strong and F2 poor and *strong*. F1 moves first and can play U (unlawful) or L (Lawful). If F1 plays U, the game ends and the players get (-1, -0.1); the payoffs reflect the sure punishment giving -1 to F1 and -0.1 to F2. If F1 plays L, then F2 could play U or L with the payoffs as shown. If F2 plays U, the game ends and the players get (-0.1, -1); the payoffs reflect the sure punishment giving -0.1 to F1 and -1 to F2. If F2 plays L, the game ends with both players getting (1, 1).

²³ Notice the engineering part of Aristotle’s work reflected in the title of this paper. The mixed constitution installs the rich and poor as sovereign, and justness is attained by *instituting* laws.

It is easy to check that (L, L) is the subgame perfect equilibrium. That means that the laws are obeyed and the constitution is stable.²⁴

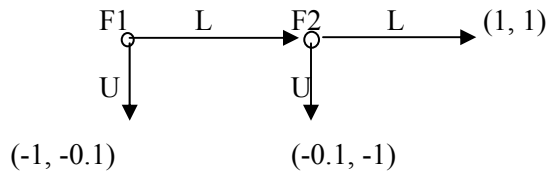


Figure 1. Under just laws of a mixed constitution, the subgame perfect equilibrium is (Lawful, Lawful) and so the constitution is stable.

The above example not only illustrates stability arising from the obedience of the laws, but also shows the linkage between conflict and laws. As said in the introduction, conflicting self-interests sustain laws that constrain self-interest to advance common interest.

Note well what the laws do. *In a world of perfect information, precise and detailed laws classify all actions as either lawful or unlawful.* Any lawful action is in common interest. That is so by definition. From the perspective of stability, it does not matter which of the lawful actions the rulers take. Stability follows from the obedience of the laws. That is also by definition. The problem is that the laws are seldom precise and their enforcement seldom perfect. So in a world of imprecision and imperfection, when can the *rulers* be expected to obey just laws? Or, when can a society install just laws as sovereign?

Imperfect enforcement of laws

“[T]he laws enunciate only general principles (AP, 3.15, 1286a7, p.221).” Indeed, “among the matters which cannot be included in laws are those which are the subjects of deliberation (AP, 3.16, 1287b15, p.

²⁴ As an example of an unstable constitution, it is easy to construct a game in which Alexander the Great will not obey democratic laws.

228).” So “officials ought to have sovereign power to act in all those various matters about which the laws cannot possibly give detailed guidelines; for it is never easy to frame general regulations covering every particular (AP, 3.11, 1282a41, p. 206).”

But officials, as noted before, are fallible not only in their judgments (as in the first inquiry) but also in their intents (the second inquiry). As Aristotle states, “it is impossible to give true judgment when their own interests and own feelings are involved (AP, 3.16, 1287a32, p. 227).” So assume that the competing factions agree to institute laws coupled with penalties to induce the members of the society to obey *both written and unwritten* laws and the specific rulings that flow from these laws. To be precise, suppose the Axiom of Mixed Constitution is satisfied.

The Axiom of Mixed Constitution. For the mixed constitution, there *exist enforceable laws* that are (a) just and (b) induce *rulers* to act lawfully with an average probability greater than 0.5.

Note that the axiom is about the laws associated with a mixed constitution, and not about the constitution itself. Constitutions can be constructed by mixing factions in any way one likes; it is akin to a thought experiment. There is no axiom required for that. The real action is in developing and enforcing laws that will support and stabilize the chosen constitution. Thus, there must exist “lawgivers” (e.g., Kleisthenes of Athens or James Madison of the United States) to develop laws to suit the constitution and there must exist the (military and police) power to enforce the laws. As the quote at the beginning of this paper suggests, having a just constitution may be desirable (normative), but without the power to enforce its associated laws, the constitution would be unobtainable (positive). “The task confronting the lawgiver, and all who seek to set up a constitution . . . , is not only, or mainly, to establish it, but rather to ensure that it is preserved intact. (Any constitution can be made to last for a day or two.) AP, 6.5, 1319b33, p. 373.”

Part (a) of the axiom says that there exists a set of laws that, *if obeyed*, would advance the common interest. Part (b) of the axiom says that there exists enough power that would cause the rulers to act as per the intent of the laws: not perfectly, but enough so that the average probability of being lawful

exceeds 0.5. *This is the probability after taking into account both the errors of judgment and intent.* Part (b) is consistent with the idea that temptation for transgression is alive as the enforcement of the laws is imperfect.

Underlying the axiom is a simple idea although modeling it is beyond the scope of this paper. Both the rich and poor factions know that the mixed constitution, with its associated laws, is an agreement for power sharing consistent with each faction's capacity for violence; recall Aristotle's laws are akin to constitutional laws of today. Thus members of each faction are deterred from acting unlawfully during peacetime by the risk of subsequent mutiny by the other faction, prolonged disruption of civic life, flight of capital and the like. They would not know what will become of them even if they survive a disrupted constitution. Moreover, legal, economic, social, institutional and informal systems create incentives to act lawfully. These may take the form of penalties, repeat play, the degree of equality, the media, norms, morals, transparency of individual actions, the cost of re-coordination to a new constitution, and so forth. Probably the history of overthrow of oligarchies and democracies would prompt rulers to be lawful; see Example 2 in section 2 where after the failure of the extreme democracy, ancient Athens returned to the mixed constitution as a compromise among the factions. Finally, the assumption that individuals act lawfully, with an average probability greater than 0.5, is the minimum required. If the average probability were less than 0.5, the opposite of Theorem 2 would happen.

Theorem 2 (Aristotle's Second Inquiry)

Consider a body of rich and poor rulers with a mixed constitution and associated just laws. A ruler can do one of two things: act as per the laws, or transgress. Suppose at equilibrium each ruler acts independently and lawfully such that the average probability of so doing exceeds 0.5. Then the probability that a majority of rulers acts lawfully will approach 1 as the size of the body approaches infinity.²⁵

²⁵ Note that the Axiom of Mixed Constitution is embedded in Theorem 2.

No proof is required because Theorem 2 is framed to match part (b) of Condorcet's jury theorem. Theorem 2 states that under the specified conditions, the mixed constitution will be *stable* (the laws will be obeyed) for a sufficiently large body. Its remarkable feature is collective near-perfection obtained from individual imperfection. It acts as some sort of a machine: drive the average probability above 0.5, and the theorem (or the law of large numbers underlying part (b) of Condorcet's jury theorem) will take care of the rest. It states that neither the mixed constitution nor its associated laws would be changed unlawfully by a sufficiently large voting body provided that the average probability remains above 0.5. In other words, *even if each citizen is corruptible, but not too corruptible on an average, a majority of a large body would be virtually incorruptible*. In Aristotle's words, "a crowd is on numerous occasions, actually a better judge than one man, whoever he might be. Again the many are less easily corrupted. As a larger amount of water is less easily polluted, so the multitude is less easily corrupted than the few. The judgment of one man is bound to be corrupted if he ... has very strong feelings about something (AP, 3.15, 1286a21, p. 222)." The foregoing is Aristotle's response to this question: "When the law either cannot decide at all or will only decide badly, ought the power to rule to rest with the one best man or with all (AP, 3.15, 1286a21, p.222)?" Theorem 2 says that conformity with the laws will be greater under majority-rule of *many* than under that of a few (or one).

Remarkably, Kleisthenes of Athens, Example 1 in section 2, installed a Council of Five Hundred for a small population of about 30 to 40 thousand Athenian voters; thus, on an average, each Council member represented no more than 80 voters (Hansen, 1991). The *largish* Council, with its members drawn from the rich, middle and poor sections, seems consistent with the size advantage offered by Theorem 2.²⁶

²⁶ Kleisthenes, who preceded Aristotle by 150 years, required joint exercise of political power, with co-equal branches of government, and an elected Council of 500 acting as the agenda setter for the People's Assembly, with the Areopagus acting as "the most revered court of law (Hansen, 1991, p. 89)".

Kleisthenes installed an extraordinary system of checks and balances, ostracism being the most famous.

The abstract of the paper says that the mixed constitution offers the best prospect to deliver both justice and stability. The assertion is true because monarchy, aristocracy and polity are unstable, and tyranny, oligarchy and democracy are unjust. In contrast, mixed constitution is both just and stable. Of course, there are many mixed constitutions depending on how democracy and oligarchy are mixed. Aristotle distinguishes between two mixed constitutions of note and these will be taken up shortly.

Aristotle on Theorem 2

Is the content of Theorem 2 implicit in Aristotle's writings? Most, if not all, of Books 5 and 6 of *The Politics* convey the essence of Theorem 2. In these books, Aristotle argues, with supportive examples, why constitutions are overthrown and how they might be preserved. Theorem 2 echoes Aristotle's main point: constitutions (of all kinds) are more likely to be overthrown as the rulers turn to extremism, and more likely to be preserved as the rulers remain moderate. In particular, oligarchies and democracies *could* become more stable by embracing moderate rather than extreme laws. To see this in light of Theorem 2, suppose the wealthy contemplate choosing between just (or moderate) and oligarchic (or extreme) laws. Clearly, the poor are more likely to obey just than oligarchic laws. That is, the average probability is more likely to exceed 0.5 if the rich adopt just rather than oligarchic laws. Hence, the wealthy would do better in the long run by adopting just laws: an oligarchy conducted on the basis of just laws would be more stable. And that is precisely what Aristotle prescribes. The logic applies to the poor as well: a democracy conducted on the basis of just rather than democratic laws would be more stable.

The main assumption of Theorem 2 is that the average probability of lawful behavior must exceed 0.5. Aristotle says so too. "In general, all those legal provisions which we say are advantageous to the constitution in each case, all these are constitutional safeguards, including that oft-mentioned and

According to the eminent Greek scholar Hansen (1991, p. 319) "never before or since has such an elaborate network of institutions been created and developed in order to run a quite small and fairly simple society."

most important principle – to ensure that the number of those who wish the constitution to be maintained is greater than that of those who do not (AP, 5.11, p. 330, emphasis added).” When the number of potential rulers who wish the constitution to be maintained is greater than that of those who do not, what can be said about the probability that a randomly drawn ruler acts lawfully? The answer is that a randomly drawn ruler would act lawfully with probability greater than 0.5.

For Theorem 3, which is closely related to Theorem 2, I introduce mixed aristocracy and mixed polity as two notable mixed constitutions. A *mixed aristocracy* is a mixture of oligarchy and democracy that leans towards the wealthy, that is, its laws lean towards oligarchic laws. A *mixed polity* is a mixture of oligarchy and democracy that leans towards the poor, that is, its laws lean towards democratic laws. Obviously, a mixed aristocracy must not lean too far towards oligarchic laws, and a mixed polity must not lean too far towards democratic laws; if they did, then the laws will cease to be in common interest. So assume that both mixed aristocracy and mixed polity fulfill the axiom of mixed constitution. Would a mixed aristocracy be more stable than a mixed polity or would it be the other way round? You would be correct if you said that a mixed polity would be more stable. The reason is that the poor are more likely to obey the laws of a mixed polity than those of a mixed aristocracy, the former being more favorable to the poor. In contrast, the rich are more likely to obey the laws of a mixed aristocracy than those of a mixed polity. But the number of poor usually far exceeds the number of rich. Therefore, the *average* probability of lawfulness will be greater under mixed polity than that under mixed aristocracy. With reference to Theorem 2, it can be shown that if the increase in the average probability is large enough, then a majority under a mixed polity is more likely to be lawful than a majority under a mixed aristocracy. Formally, we have Theorem 3.

Theorem 3

Consider a sufficiently large body of rulers under two regimes: mixed aristocracy and mixed polity. Each ruler can do one of two things: act as per the laws of the regime in force, or transgress. Suppose at

equilibrium each ruler acts *independently* and lawfully such that the *average* probability of so doing is p_{avg} under mixed polity, and a_{avg} under mixed aristocracy. Suppose both averages exceed 0.5 and that p_{avg} is sufficiently greater than a_{avg} . Then the probability that a majority of rulers acts lawfully under mixed polity will exceed the corresponding probability under mixed aristocracy.

Proof. See Appendix.

Theorem 3 says that if the average probability of individual lawfulness under mixed polity is sufficiently greater than that under mixed aristocracy, then mixed polity would engender greater lawfulness than mixed aristocracy. Hence, a mixed polity is likely to be more stable than a mixed aristocracy.

It is time to compare notes with Aristotle. Aristotle states that how oligarchy and democracy are mixed “makes some more and others less stable (AP, 5.7, 1307a5, p. 320).” “Those that lean rather towards oligarchy are called aristocracies, those that lean towards the mass of the people are called polities; and *the effect is to make polities safer than aristocracies*. This is due to the fact that greater size means greater strength; for where people have equal shares, they are more content, but those who have the advantages of riches, if they enjoy a preponderance in the constitution, seek to ill-treat others and enhance their own fortunes (AP, 5.7, 1307a5, p. 320, emphasis added).” Aristotle not only captures the essence of Theorem 3, but also explains why it works. And it is this conclusion of Aristotle that Dahl (1989) misses in “Democracy and Its Critics” as explained in section 2.

A clarification is in order. In the above quote, Aristotle’s polity is this paper’s mixed polity, and Aristotle aristocracy is this paper’s mixed aristocracy. Aristotle’s use of polity and aristocracy, first as a just constitution of good men, and then as a mixture of oligarchy and democracy, is confusing.²⁷ I

²⁷ As I see it, after using the word polity as a constitution of many good men for the theoretical purpose of the first inquiry, Aristotle concluded that many men cannot be expected to be good: a polity (of good men) cannot exist. This freed the word polity and Aristotle used it again this time to denote a mixture of oligarchy and democracy.

have tried to avoid the confusion by using the word polity in Aristotle's first inquiry, and the expression mixed constitution in the second inquiry; Aristotle uses the word polity in both inquiries. I have used mixed polity and mixed aristocracy (instead of Aristotle's polity and aristocracy) to distinguish between two mixed constitutions.

6. The middle class as a stabilizer of the mixed constitution

Theorem 2 states that a majority of rulers under a mixed constitution would offer both justice and stability provided that the average probability that each individual acts lawfully exceeds 0.5. Not surprisingly the average probability must exceed 0.5 at all times. Should the average probability fall below 0.5 at some point, a majority of a large body would almost surely act unlawfully. Thus, if orators, wars, short-term opportunities cause enough people to be swayed from just laws, then a majority of rulers would end up doing great harm to the society. Theorem 3 states that a mixed constitution should lean towards the masses so that the average probability exceeds 0.5 with some margin to spare. Yet history is filled with instances where mixed constitutions collapsed due to internal rift. Indeed, Plato witnessed, through much of his youth, two polar factions, rich and poor, in constant struggle. And Athens, between 460 and 400 BCE, was bedeviled by majority tyranny and minority coups offering neither justice nor stability.

The role of the middle class is articulated by Euripides (*The Suppliants*, 420 BCE): "There are three classes of citizens: the rich are useless and always lusting for more; the poor, who lack their daily bread, are dangerous, for they assign too great a place to envy and hurl their stings at the rich, being deceived by the tongues of the wicked leaders; of the three classes the one in the middle preserves states by keeping to the discipline that the city establishes (Euripides, 1998, p. 39)."

Aristotle picks up the Euripides theme and argues that a large middle class will reduce the temptation for the rich and poor to usurp power opportunistically. According to Sinclair (AP, p. 254), a commentator and translator of *The Politics*, "Aristotle sees an inverse ratio between the amount of property possessed by the politically dominant part of the state, and the rule of law: the greater the

property, the greater the opportunity for that part to seize the reins of power for itself, and the less willingness to let the law rule.” In other words, high inequality produces a dominant faction and unjust laws.

A large middle class, because it enhances equality, would enhance the prospect of the rule of law. Politically, a large middle class acts as a buffer between the rich and the poor, *has enough stake in its holdings* that it would guard against demagogues and is sufficiently powerful to guard against the oligarchic concentration of power. Aristotle asserts, “The best partnership in a state is the one which operates through the middle people ... For the addition of its weight to either side will turn the balance and prevent excess at the opposite extremes (AP, 4.11, 1295b34, pp. 267-8).” Aristotle continues: “Whenever the middle people outweigh a combination of the two extremes, or even one only, then there is a good chance of permanence of the constitution. There is no danger of rich and poor making a common cause against *them*; for neither will want to be slaves to the other, and if they are looking for a constitution more acceptable to both, they will not find any better than this. Their mistrust of each other would make it impossible for them to accept alteration in office (AP, 4.12, 1296b34, p. 272).”

Let us examine the middle class in light of Theorems 2 and 3. The above discussion suggests that the interest of the middle class is close to being the common interest. For example, a member of the middle class would like to provide for the poor in part because he or she might end up there, and also for the rich in part because he or she strives to be rich. In a sense, a member of the middle class is sitting behind the “veil of ignorance:” she or her off-springs might end up as rich, poor or remain in the middle. Therefore, her actions would be just: she would pursue the well-being of the rich, middle and poor; a good deal of it in self-interest. With this in mind, imagine a mixed constitution comprising the rich, middle and poor. The laws that this constitution will adopt will probably be close to the ideal of the middle class. As a result, *the middle class will be lawful simply by acting in its self-interest.*

Assume that the members of the middle class act in self-interest, that is, lawfully with probability close to 1. If the middle class is large enough, then the average probability that a citizen acts

lawfully will exceed 0.5 with sufficient margin. The margin will be enough to absorb short-term fluctuations in the average probability while remaining above 0.5 at each instant.²⁸ Aristotle adds “[a] constitution of the middle people is nearer to democracy than is a constitution of the few, and is of all such constitutions the safest (AP, 5.1, 1302a2, p. 299).” A sizeable middle class has the effect of tilting a mixed constitution towards democracy. In other words, a sizeable middle class has the effect of tilting the mixed constitution towards a mixed polity which is among the safest of all mixed constitutions; see Theorem 3.

7. Conclusion

Kraut’s remark that *The Politics* is “nothing more than a collection of essays on diverse political topics” as well as Barker’s (1978, p. 157, fn 2) comment that “Aristotle, in different contexts, says different things, which cannot be easily reconciled” are addressed. This paper has attempted to show that *The Politics* takes a clear path to a clear destination: a just and stable constitution. The paper shows that *The Politics* is written with enough precision that three theorems emerge from it. The first theorem, which is the final result of Aristotle’s first inquiry, establishes Aristotle’s conjecture that when men are all good, a polity is better than an aristocracy and an aristocracy is better than a monarchy. The second theorem, which is a part of Aristotle’s second inquiry, shows that a mixed constitution, with its associated just laws, would be stable so long as the enforcement of laws is sufficiently good that the average probability of individual lawfulness exceeds 0.5. The third theorem, again a part of Aristotle’s second inquiry, shows that a mixed constitution that leans towards democracy would be more stable than a mixed constitution that leans towards oligarchy. Theorem 3 proves Aristotle’s literal assertion to that effect. Moreover, the Euripides-Aristotle hypothesis that the middle class helps establish a just and stable constitution follows from the statements of Theorems 2 and 3. Finally, much of Aristotle’s writing on how constitutions are

²⁸ Ideally, a dynamic model is required to capture fluctuations over time. But that is beyond the scope of this paper.

destroyed and how constitutions are preserved can be linked to the average probability of individual lawfulness. Raise the average probability by implementing moderate laws or improving education (education being a theme of Books 7 and 8 of *The Politics*) and the constitution is more likely to be preserved. Lower the average probability by implementing extreme laws and the constitution is more likely to be destroyed.

Should we be surprised by the depth of Aristotle's *Politics*? I think we better. It is a comprehensive document written long ago. It applied in ancient times and its theorems apply to today's world. The great, and probably unappreciated, strength of *The Politics* rests in its economic or game-theoretic approach: it is fully alert to self-interest, the role of conflict and sanctions in the enforcement of laws, checks and balances, and the law of large numbers. Moreover, it is not dependent on "extreme" rationality. It is compatible with the idea of satisficing. Thus, if the average probability crosses the threshold of 0.5 due to the fear of sanctions, goodness of citizens or better education, it makes no difference.

Diverging from Plato's assertion that stability is justice, Aristotle treats them as different. Moreover, Aristotle's *Politics* emerges as a document in defense of democracy (= mixed constitution) whereby, speaking non-poetically, democracy (= mixed constitution) is a government by the rule of law that offers justice for all. Moreover, the conduct of democracy includes not only majority-rule voting but also power sharing so that factions prefer legitimate political contests to violence.

Table 1
A summary of the topics covered by *The Politics*

Book 1	On state as an association, on state and individual, households, slaves, and on monopoly.
Book 2	Criticisms of Plato's Republic and Plato's Laws, a review and criticism of the constitutions of Phaleas, Hippodamus, Sparta, Crete and Carthage, a review of certain lawgivers.
Book 3	Ideas of a citizen, state, good man, and correct and deviated constitution; definitions of democracy and oligarchy; the questions of the just distribution of political power (ideas of democratic, oligarchic and absolute justice), justice and sovereignty (where should the power of the state reside?), the wisdom of collective judgments; the idea that just laws <i>ought</i> to be sovereign; search for the best constitution; five types of kingship.
Book 4	Constitution and laws defined; systems must be incentive compatible; why there are several constitutions; four type of democracies and four types of oligarchies, varieties of aristocracy and tyranny; polity (I call it <i>mixed</i> polity in the second inquiry: a mixture of democracy and oligopoly that leans towards democracy) distinguished from aristocracy (I call it mixed aristocracy in the second inquiry: a mixture of democracy and oligopoly that leans towards democracy); what destroys and what preserves constitutions; merits of the middle constitution: why democrats and oligarchs should cultivate the middle ground; deliberative (legislative), executive and judicial elements of the constitution.
Book 5	Sources of constitutional change, why democracies and oligarchies are overthrown (because they ill-treat the opposing faction, demagogues treat the wealthy unjustly, oligarchs wrong the multitude, the courts are not manned by the citizen-body, etc.); mixed constitution are overthrown because of improper mixing of oligarchy and democracy; how constitutions may be preserved (proper treatment of citizens and non-citizens, avoid centralization of power, greater income equality, greater moderation, give equality to others to the rich in democracy and to the poor in oligarchy etc.), the impermanence of tyrannies.
Book 6	How do constitutions function best? Ways of achieving equality. How to preserve a democracy? How to preserve an oligarchy?
Book 7	Virtue and prosperity, on active life, on the size, territory, environment, food supply, and defense of the state; on temples, on markets, on education.
Book 8	On education of various types.

Table 2

Focus: Factional Interest (Unjust)

Focus: Collective Interest (Just)

<p><i>Democracy</i> is a constitution of the poor (the citizens are poor), by the poor (of imperfect virtue pursuing factional interest under <i>unjust laws</i>), and for the poor.</p>	<p><i>Polity</i> is a constitution of the poor, by the poor (of perfect virtue pursuing common good under <i>just laws</i>), and for the <i>people</i> (for the benefit of all factions).</p>
<p><i>Oligarchy</i> is a constitution of the wealthy (the citizens are wealthy), by the wealthy (of imperfect virtue pursuing factional interest under <i>unjust laws</i>), and for the wealthy.</p>	<p><i>Aristocracy</i> is a constitution of the wealthy and virtuous (the citizens are people of merit), by good men (of perfect virtue pursuing common good under <i>just laws</i>), and for the <i>people</i> (for the benefit of all factions).</p>
<p><i>Tyranny</i> is a constitution of one man, by one man (of imperfect virtue pursuing <i>his</i> own interest under), and for one man.</p>	<p><i>Monarchy</i> is a constitution of one good man, by one good man (of perfect virtue pursuing common good), and for the <i>people</i> (for the benefit of all factions).</p>
<p>A <i>mixed constitution</i> is that of the competing factions (oligarchs and democrats), by the competing factions (<i>each individual</i> of imperfect virtue pursues <i>factional interest subject to just laws</i>), and for the competing factions.²⁹</p>	

²⁹ Observe that the Athenian slaves were not a faction organized to compete for power and are thus not a part of the mixed constitution. A mixed constitution (democracy in the modern sense) is primarily for the politically organized.

Appendix

Proposition 1. Consider a voting body such that its members vote independently. At equilibrium, suppose voter i votes for the better alternative with probability p_i for all i . Denote the probability that majority votes for the better alternative with P_A . Then for a sufficiently large voting body, $P_A(p_i$'s are not all equal) $>$ $P_A(p_i$'s are all equal). That is, diversity improves the voting performance.

Proof. For $i = 1, \dots, n$, let $X_i = 1$ if i votes for C and $X_i = 0$ if i votes for F, where C represents common interest and F represents factional interest. Suppose $\Pr(X_i = 1) = p_i$ for all i . Let $p = (p_1 + \dots + p_n)/n$ and $X = (X_1 + \dots + X_n)/n$ be the averages. It is to be shown that $\Pr(\text{majority votes for C} \mid p_i$'s are not all equal) $>$ $\Pr(\text{majority votes for C} \mid p_i$'s are all equal).

By the definition of X , $\Pr(\text{majority votes for C}) = \Pr(X > 0.5)$. So need to prove that $\Pr(X > 0.5 \mid p_i$'s are not all equal) $>$ $\Pr(X > 0.5 \mid p_i$'s are all equal). With $(X_1 + \dots + X_n)$ being binomially distributed, the distribution of X can be approximated by normal distribution for sufficiently large voting bodies. Observe that the expected value of X is p whether the p_i 's are equal or not. So the proof will be complete if $\text{Var}(X \mid p_i$'s are not all equal) $= [p_1(1-p_1) + \dots + p_n(1-p_n)]/n^2 <$ $\text{Var}(X \mid p_i$'s are all equal) $= p(1-p)/n$. Now the left-hand side $= [np - (p_1^2 + \dots + p_n^2)]/n^2$. The sum $(p_1^2 + \dots + p_n^2)$ attains its minimum, and hence the variance on the left-hand side attains its maximum, when all the p_i 's are the same (Feller, 1968). Thus, the desired inequality holds.³⁰ This completes the proof.

Theorem 3

Consider a sufficiently large body of rulers under two regimes: mixed aristocracy and mixed polity. Each ruler can do one of two things: act as per the laws of the regime in force, or transgress. Suppose at equilibrium each ruler acts *independently* and lawfully such that the *average* probability of so doing is

³⁰ To illustrate, let $p_1 = .6$, $p_2 = .8$. Then $p = .7$. So $V(X \mid p_i$'s are not all equal) $= [p_1(1-p_1) + p_2(1-p_2)]/2^2 = (.24 + .16)/4 = .10 <$ $.21/2 = V(X \mid p_i$'s are all equal $= .7) = p(1-p)/2$.

p_{avg} under mixed polity, and a_{avg} under mixed aristocracy. Suppose both averages exceed 0.5 and that p_{avg} is sufficiently greater than a_{avg} . Then the probability that a majority of rulers acts lawfully under mixed polity will exceed the corresponding probability under mixed aristocracy.

Proof. Suppose *momentarily* that each citizen acts lawfully with the same probability $p > 0.5$. Then it is easy to show that the probability that a majority acts lawfully is strictly increasing in p .³¹ Letting $p = p_{avg}$ and $a = a_{avg}$, it follows that if $p > a > 0.5$, then $P_p(p) > P_a(a)$ where $P_p(p)$ and $P_a(a)$ denote the probabilities that a majority acts lawfully under mixed polity and mixed aristocracy, respectively. Thus, a mixed polity would do better than a mixed aristocracy.

Now suppose that all individuals do not act lawfully with the same probability. This creates a problem because it is possible that a *small* rise in the average probability lowers the probability of majority lawfulness; illustrated in the next footnote.

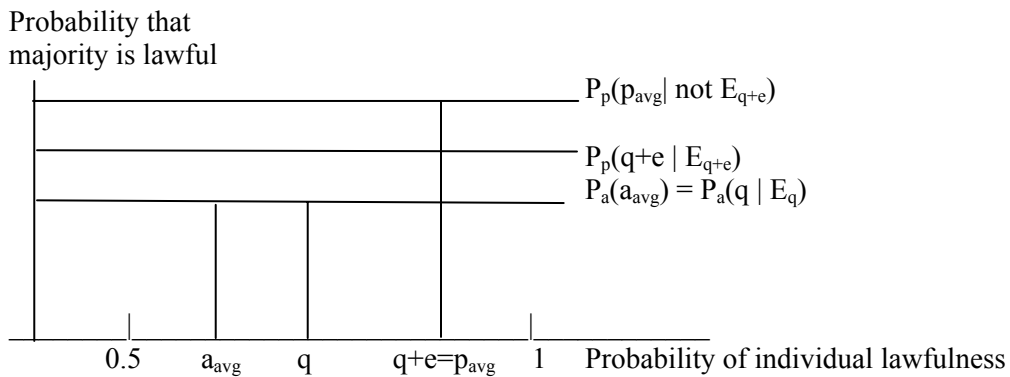


Figure 2

Let q be a number in $(0.5, 1)$ such that the probability of majority lawfulness for *aristocracy* $P_a(a_{avg}) = P_a(q | E_q)$, where E_x denotes the event that each citizen acts lawfully with the same probability x . Such a q will exist by continuity. See Figure 2. Then q will be greater than a_{avg} . That is because if $q = a_{avg}$, then

³¹ Expand the probability that a majority acts lawfully using binomial distribution and then show that its derivative with respect to p is positive.

by Proposition 1, $P_a(a_{\text{avg}}) > P_a(q | E_q)$ which would contradict the definition of q ; if $q < a_{\text{avg}}$ then once again $P_a(a_{\text{avg}}) > P_a(q | E_q)$. It is understood that $P_a(a_{\text{avg}})$ is the probability of majority lawfulness when the average probability is a_{avg} and *not all act with the same probability*.

Consider the number $q + e$ where $e > 0$ such that $q + e < 1$. Then, from the first part of this proof, $\Pr(a \text{ majority acts lawfully} | E_{q+e}) > \Pr(a \text{ majority acts lawfully} | E_q)$. Set $p_{\text{avg}} = q + e$ for $e > 0$ and small at pleasure. Then it follows from Proposition 1 that $P_p(p_{\text{avg}}) > P_a(q + e | E_{q+e})$; note that the use of p_{avg} signifies that we are talking about the polity on the left-hand side. Moreover, it is understood that $P_p(p_{\text{avg}})$ is the probability of majority lawfulness when the average probability is p_{avg} and *not all act with the same probability*.

Putting things together, we have $a_{\text{avg}} < q < q + e = p_{\text{avg}}$. It follows that if p_{avg} is sufficiently greater than a_{avg} (enough to embed the number q in between), then $P_p(p_{\text{avg}}) > P_a(q + e | E_{q+e}) > P_a(q | E_q) = P_a(a_{\text{avg}})$, that is, $P_p(p_{\text{avg}}) > P_a(a_{\text{avg}})$. That is, the probability that a majority of rulers acts lawfully under mixed polity will exceed the corresponding probability under mixed aristocracy. This completes the proof.³²

³² Note that a rise in the average probability does not necessarily raise the probability of majority lawfulness. To see this, start with $P(p_{\text{avg}}) > P(q + e | E_{q+e})$ where $p_{\text{avg}} = q + e$; the subscripts a and p are ignored because they are not relevant for the point to be made. Then by continuity, there exists a number $f > 0$ such that, $P_p(p_{\text{avg}}) > P_a(q + e + f | E_{q+e+f})$. Observe that a rise in the average probability to $q + e + f$ from p_{avg} is accompanied with a drop in the probability of majority accuracy to $P_a(q + e + f | E_{q+e+f})$ from $P_p(p_{\text{avg}})$.

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