Econophysics: Comments on a Few Applications, Successes, Methods and Models

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Abstract

For this special issue, the article aims at discussing a few econophysics problems studied so far rather successfully. The following 'applications' in micro-econophysics are considered: (i) financial crashes; it is emphasized that one can distinguish between endogenous and exogenous causes; (ii) portofolio control, selection and inherent risk measure; (iii) foreign currency exchanges, also distinguishing endogenous and exogenous money control; (iv) price and asset evolution values. It is shown that some macro-econo-physics problems have been also tackled, like geographic/political constraints, the globalization of the economy and country clustering. Moreover, it is daring to suggest prospect for studies and researches, whence presenting some selection of a few interesting perspectives.

Keywords

Micro- and macro-econophysics, financial crash, risk control, foreign currency exchange, economy globalization, endogenous and exogenous causes, price and asset evolution

> If a physicist wants (= tries!) to sell the solution of a problem to an economist, he (the physicist) must pay a (the ?) price.

Introduction

Does Econophysics Make Sense? asks Stauffer in February 1999 (Stauffer, 1999). Econophysics is a recent branch of physics that applies Statistical Physics methods to problems concerned with financial features and with practical problems in micro-and macro-economics. As early as 1693, at the econo-statistician level, Halley had showed how detailed records of births and deaths, in Breslau, now Wroclaw, PL, could be used to compute life expectancies and published the first detailed mathematical analysis of the valuation of annuities (Ciecka, 2008; Halley, 1693). Newton (Cohen, Whitman & Budenz, 1999) and Gauss also theorized financial speculation. Newton's financial investment losses (in the South Sea bubble burst) are well reported (Gleick, 2004; Westfall, 1994). Gauss was much more successful (Hall & Froderberg, 1970).

Bachelier, a mathematician, developed a 'theory of speculation', in his 1900 PhD. thesis (Bachelier, 1900). Bachelier's work suggests a practical connection between stochastic theory (random walks or Brownian motion) and financial analysis. His theory, for option pricing, was received with extreme skepticism at the time (even Poincaré was rather critical about his student's views). Forgotten for more than half a century, Bachelier's work is nowadays considered as a milestone in Econophysics (Mantegna & Stanley, 1999). Another milestone is Mandelbrot's 1963 observation of power-law scaling in commodity markets (Mandelbrot, 1963); see also Mandelbrot (1967).

Of course, practical economists have been running computer simulations of financial markets models for a long time. However, one can admit that models in modern quantitative financial analysis were stimulated by the work of physicists, for example, Osborne (1959) who in the 1950s rediscovered the Brownian motion signature in stock market dynamics.

A major achievement in the field was the early 1970s work of Black and Scholes (1973) and of Merton (1973) whose model casts the option pricing problem (Merton, 1990) into a diffusion type equation, most often referred to today as the Black–Scholes equation. It can also

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be embedded in a quantum physics setting, as shown by Haven (2002). However, it was to physicists to show that the Black–Scholes equation is based on erroneous hypotheses (Bouchaud & Sornette, 1994).

To make it short, physicists are now applying concepts from Statistical Physics, such as, the ideas of universality, scaling, game theory, networks and agent interaction models, to practical financial issues, such as, financial crashes, the movement of stock prices and portfolio management, currency exchange rate fluctuations, share price evolution, etc. Moreover, many recent attempts turn towards contributions to the theory of macro-economics. Papers now being published span wider areas, such as, consumer choice (Lambiotte & Ausloos, 2005, 2006) and social trends (Della Vigna, List & Malmendier, 2012), business cycles (Ausloos, Miśkiewicz & Sanglier, 2004) and the (structural and financial) evolution of organizations (Ausloos, 2012; Ross & Ausloos, 2009), which are also of interest to social scientists and managers.

It is aimed here in the following to discuss a few problems, tackled in econophysics, having in mind a readership of students/teachers of economics and business management level, wishing to be aware of questions raised by economists and somewhat understood by physicists interested in applying usual methods of investigations and modelling techniques to such economy and finance problems. Only a few econophysics problems studied so far are presented in the following—the selection being surely very biased, since it focuses on the author's activities and interests.

Thus, after this introductory section, suggesting the immense work field available for investigations, a few cases are outlined in the second section 'Micro-Econophysics', first stressing financial crashes in part A of the second section. It is emphasized that 'external fields' are a major ingredient in order to distinguish endogenous from exogenous causes, rendering the study more (or very) 'interesting', within a mechanistic approach based on parity and symmetry of concepts. In part B of the second section, portofolio selection and inherent risk are shown to have been considered.

Foreign exchange markets have been also much studied—the money volumes being very large, whence also gains and losses. Several features are discussed in part C of the second section. Finally, predictability is, at a very general level, a major concern of agents, being traders, investors, managers or politicians. The point is discussed in part D of the second section. Finally, a comment on price evolution is necessary (see self-citation at the beginning of

the main text). It is presented in part E of the second section with some introduction to models on asset distributions.

In the third section 'Macro-Econophysics Aspects', considerations more prone to macro-economy questions, like the globalization of the economy or (country) economic clusters, are shown to have been considered in recent econophysics work.

A few short comments, in the fourth section 'Perspectives and Prospects', are tied to daring suggestions and prospects for studies and researches, while presenting some selection of interesting (?) perspectives.

A few Appendices contain technical details which can be found with much more information in many books or other publications. Too many to quote them here! A short conclusion is found in the fifth section.

However, to close this introduction, and put the main section into a proper perspective, let it be mentioned that much data analysis work in econophysics is based on

- rank-size, frequency-size concepts, and usual statistical tests and considerations,
- analysis of the correlation functions of (often the fluctuations in) financial signals,
- …often (alas) equal time correlation functions, though various time scales, and time lags should be much more often considered.

Next, economic/financial models are constructed based on developments/generalizations of basic physics models (Brownian motion, Ising and Potts or ferroelectrics models) modernized into agent based models, often studied on networks.

Finally, two basic techniques are used for finding 'solutions' or features, namely,

- analytic work, studying one or a set of differential equations,
- numerical simulations, often using the Monte-Carlo method.

Again, let it be emphasized that the note is mainly based on a few of *my* contributions, thereby apparently much reducing the scope of the many existing and brilliant investigations. This is not a statement of contempt; on the contrary, it implies an accent of much modesty.

Micro-econophysics

There is much data available on financing, banking, option markets, stocks and currency exchange rates, discount and

interest rates. There are many levels of observation: individual income, individual expenses, checking accounts and savings, public or private accounts, volumes, debts and credits, tellers, dealers, bank outlets, businesses, governments. One is immediately tempted to play *statistics*, expecting to reach some stylized features of financial matters.

What should be the number of data points? Physicists prone to thermodynamics and *statistical physics* have used from the very beginning the Avogadro number as an upper limit. The lower limit is always prone to criticism. Yet, one should remember that there are appropriate tests (Student $t-$, χ^2, ...). These can be sufficient, when the data has been taken on some kind of *equilibrium state*.

It is worth to point out that there is also no drastic need for considering a large phase space in order to describe complex *non-equilibrium systems*. Since Lorenz (1963), at least, one knows that *three* coupled nonlinear differential equations can lead to the prediction of steady states, cycles and deterministic chaos—in a *continuous* time approximation. At the *discrete* time level, since Feigenbaum (1980), one knows that a condition, implying the third derivative of the one-*dimensional* mapping, informs on the possibility of chaotic systems.

Thus, there is no need for introducing irrationality or stochasticity, from the first steps, to describe complicated evolutions. Some simplicity is already leading to very complicated features when nonlinearity is implied. Moreover, the phase space (the number of variables) may always be reduced a lot. Of course, economists can always claim that 'this is a too much reduced phase space'. Indeed. What is then the relevant size?

Clearly, one can always introduce new variables and evolution equations (Asada & Ouchi, 2013). But one can, thereafter, also argue that these variables and equations do not take into account, for example, (non-exhaustive list) the cultural, sport and 'social', aspects of the economic society—though they are very relevant nowadays! However, simplicity and intuition are recommended at the starting gate. Results should be convincing if they recover stylized facts, besides implying forecasting. Nevertheless, the question of the size of the relevant phase space in *micro-economy* problems should be a relevant study, and some worthwhile information for further econophysics studies and applications.

Let it be recalled, at this stage, that beyond equilibrium phenomena, physicists and mathematicians discovered dissipative structures (Nicolis & Prigogine, 1977), intermittency (Barabási & Vicsek, 1991) and self-organized criticality (Bak, 1996; Bak, Tang & Wiesenfeld, 1987, 1988) in non-equilibrium states. In fact, cooperative effects like those seen at phase transitions, thermodynamic ones (Fisher, 1974; Stanley, 1971) or geometric (or probabilistic) ones (Stauffer & Aharony, 1992), have been found to be analogous to those found, for example, in financial bubbles (discussed later). An essential ingredient, the fractal concepts (Addison, 1997; West & Deering, 1995) were already pointed out 'long ago' (Peters, 1994). This is one of the origins of physicists' competition to overcome mathematicians who have invaded economy faculty or arbitration rooms or brokerage firms.

Of course, statistical fits and inherent tests are useful (Ausloos, 2001), including statistical models, but parameters should be controllable and their variation understood. This leads to emphasizing the major difference between 'statistician models', like ARCH and subsequent extensions, and modern econophysics models, based on 'agent' behaviours. Physicists know the limits of any model because of inherent hypotheses and should honestly question the findings. However, the forecasting is better controlled in econophysics than through the many theories which many economists invent and discuss. One famous case is that of financial crashes predictions.

A. Crashes

One spectacular set of features belongs to financial crashes. Because of its magnitude, the stock market crash of October 1987 outshines all downturn ever observed in the past. In one day, the Dow Jones lost 21.6 per cent and the worst decline reached 45.8 per cent in Hong Kong (Bates, 1991). Compared to the most famous crash of October 1929, the crash was spread over 2 days, a 12.8 per cent on 19 October followed by a 11.7 per cent drop the next day. Note that Lauterbach and Ben-Zion (1993) found that trading halts and price limits had no impact on the overall decline of October 1987, but merely smoothed return fluctuations. Another supposedly major characteristic of the crash of October 1987 is the phenomenon of irresistible contagion that the shock started (Roll, 1989).

Some application of statistical physics ideas to the description of such stock market behaviour was proposed in Feigenbaum and Freund (1996) and Sornette, Johansen and Bouchaud (1996). They indicated that most economic indices follow a divergence-like power law with a *complex* exponent, $m = m' + i\,m''$ (see Appendix 1). A rupture occurs at $t = t_c$, the crash time. This law is similar to that of critical points at so-called second order phase transitions (Fisher, 1974; Stanley, 1971), but generalizes the scaleless situation ($m'' \equiv 0$) for cases in which discrete scale invariance exists

(Anifrani, Le Floc'h, Sornette & Souillard, 1995; Sornette, 1998). Indeed, superposed to the divergence, described by m', a series of oscillations, with frequency related to m", could be observed, as in the ionic content of water sources before some earthquakes (Johansen *et al.*, 1996), suggesting an analogy including precursors and replicas (Sornette *et al.*, 1996). Interestingly, the period of these oscillations is logarithmically reducing as one converges to the rupture point t_c, leading to the appellation *log-periodicity*. Alas, the parameter fits are not robust against small perturbations due to the numerical instability of a nonlinear seven-parameter fit.

Thus, it has been proposed that the 'universal exponent' m is, in fact, close to zero (Vandewalle & Ausloos, 1998c; Vandewalle, Boveroux, Minguet & Ausloos, 1998), that is, the divergence of a financial index $y(t)$ for t close to t_c should rather behave like a logarithmic function (see Appendix 2). In fact, this is the behaviour of the specific heat (a 'four point correlation function') of the magnetic (two-dimensional) Ising model (Fisher, 1974; Stanley, 1971). Another type of similar (so-called 'essential') singularity in physics is that occurring at the Kosterlitz–Thouless (KT) phase transition (Kosterlitz & Thouless, 1972, 1973), for dislocation mediated melting. It represents the transformation from a disordered vortex fluid state with equal number of vortices with opposite 'vorticity' to an ordered molecular-like state with molecules composed from a pair of vortices with different polarities across the KT transition. The logarithmic behaviour can also be observed in the temperature derivative of the electrical resistivity of magnetic systems at the paramagnetic–ferromagnetic phase transition (Ausloos, Leburton & Clippe, 1980; Sousa *et al.*, 1979; Sousa *et al.*, 1980). The behaviour is thus generally specific to systems with a *low order* spatial dimensionality of the so-called 'order parameter'.

Thus, one can test such a (seven or six parameter) function on financial indices, that is, on one hand, there is a four- or three-parameter divergence and, on the other hand, there is a log-periodic function describing the oscillations of the financial index before the crash (see Appendices 1 and 2).

In so doing, one could monitor many indices after 1987. In 1997, the stock market numerical conditions astonishingly looked like the pre-crash period of 1987. Thus, in view of the huge similarity between the 1987 and 1997 behaviours, one could come up with accurately predicting the 27 October 1997 crash (Dupuis, 1997a, 1997b; Legrand, 1997a, 1997b; Vandewalle & Ausloos, 1998c; Vandewalle

et al., 1998). Much discussion has followed such a prediction, often called 'accidental' by those who did not predict it (Laloux, Potters, Cont, Aguilar & Bouchaud, 1998). However, other successes can be mentioned (Ausloos, Ivanova & Vandewalle, 2002; Brisbois, Boveroux, Ausloos & Vandewalle, 2000; Johansen, 2002; Johansen & Sornette, 1999, 2000; Johansen, Sornette & Ledoit, 1999; Kwapień & Drożdż, 2012; Stauffer & Pandey, 2000). Sometimes, necessarily *a posteriori*, one can even detect 'near-crashes'; see Appendix 2.

Later on, many investigations have allowed to understand that such a behaviour, that is, a divergence and a log-periodicity of the oscillations, can arise when the systems present an endogenous discrete scale invariance, that is, the system (= market) has an inner structure which can be *mutatis mutandis* reproduced at different scales (or levels). A sandpile on a fractal-like basis is a fine analogy (Ausloos *et al.*, 2002). It suggests that such a crash is due to a limited number of agents (the most 'important/ active' ones) who trigger an avalanche. One can, therefore, consider that there are two basic categories of financial crashes: endogenous ones and exogenous ones (Johansen & Sornette, 2010; Kwapień & Drożdż, 2012; Sornette, 2006; Sornette & Helmstetter, 2003).

The exogenous crashes should appear more suddenly because they are triggered by *quasi* unexpected news, termed 'external fields' in physics. Alas, they are less easily predicted. It seems that they seem to occur more frequently than endogenous crashes. Moreover, statistical physics studies on triggered phase transitions indicate that one should expect less universal features in exogenous shocks. On this point, one should note that a study of sales, when either based on publicity or on buyer herding, indicates a different range of relaxation times (Lambiotte & Ausloos, 2006): the relaxation time seems to be twice shorter in endogenous shocks than in exogenous ones. Exogenous and endogenous after-shocks sales are thus discriminated by their short-time behaviour. Many studies in econophysics pertain to the recovery of financial indices after a crash, but much has still to be done in order to associate such simple analytic laws with the fundamental laws of economy in a broad sense. For some completeness, let the work of Drożdż *et al.* be emphasized (Kwapień & Drożdż, 2012), and their references therein.

Specifically, objective ambitions of physicists are only to analyze financial data in order to find whether a break point in the series has a precursor, and some after shock (Sornette *et al.*, 1996). In data analysis as most often done in this matter, the crash target date represents the ultimate

date of a break if the stock market indices continue their extraordinary growth. In brief, the study consists of watching if a major correction is becoming more and more imminent, even if it can take several different forms. Another test point is the crash amplitude. Finally, the crash duration, the after-shocks and replicas are also interesting questions.

To be fair, note that other features should be taken into account when predicting and describing a financial crash. Indeed, financial bubbles and crashes are associated with the phenomenon of *volatility clustering* (Cont, 2007), that is, the existence of successive periods of small and large amplitude of an index fluctuations. The volatility clustering is seen through the autocorrelation function of the index fluctuation amplitudes. Interestingly, one observes that the autocorrelation function is decreasing as a power-law, like the autocorrelation function of the order parameter, near a critical (thermodynamic) phase transition, for example, the susceptibility in magnetic systems (Stanley, 1971). Similarly, trading volume is positively correlated with market volatility, and both trading volume and volatility show the same type of long memory behaviour (Lobato & Velasco, 2000). This is understood if we consider that the market becomes more and more sensitive to perturbations, as when an order parameter is approaching the critical transition state. An arbitrarily small fluctuation due to some arbitrary agent can trigger a cascading response of the market (Lux & Marchesi, 2000). This also resembles intermittence phenomena (Bergé, Pomeau & Vidal, 1987) and is much reminiscent of *Self-Organized Criticality* (SOC), that is, the tendency of dissipative systems to spontaneously evolve into a 'critical' state (Bak, 1996).

In brief, long memory at crashes, in bubbles, is directly linked with the effect of volatility clustering, while the log-periodic substructure is due to hierarchical cascades.

B. Portfolio Selection

Risk must be expected for any reasonable investment. However, risk is an intuitive notion that resists formal definition (Holton, 2004). One expects that a portfolio should be constructed such as to minimize the investment risk in presence of somewhat unknown fluctuation distributions of the various asset prices in view of obtaining the highest possible returns (Marshall, 1994; H. Markowitz, 1959; H.M. Markowitz, 1952). Usually analysts recommend investment strategies based, for example, on moving averages, momentum indicators. Another sort of data

analysis technique can be adapted to portfolio management (Ausloos & Bronlet, 2006), leading to forecasting and prediction, with some 'risk': it is known as the Zipf technique, originating in work exploring the statistical nature of languages (Zipf, 1949); see Appendix 3. The Zipf method previously developed as an investment strategy (on usual financial indices) (Ausloos & Bronlet, 2003; Bronlet & Ausloos, 2003) can be adapted to portfolio management.

In brief, if a crash can sometimes be predicted as suggested here earlier, the next fundamental step is to predict the amplitude of the fluctuations before (and also after) the event. Indeed, this information should allow economic agents to hedge portfolios on derivative markets through call or put options. The problem reduces in fact to extract the true background contribution of an index. Indeed, it represents the 'natural' (noisy and structural) evolution of stock indices out of any euphoric ('anomalous') bubble evolution. This is a classical question in thermodynamic phase transitions before searching for critical exponents characterizing power law behaviours. This has been somewhat tackled through Zipf' 'lazy method' of merely counting *up* or *down* fluctuations, and expecting some 'only smooth non-equilibrium' of the market.

The index time series of the closing price of some stock is translated into a series of letters, for example, two alphabet characters (u, d), corresponding to *up* or *down* fluctuations. (this can be generalized, of course). One can next search for all possible words of m letters, and investigate the occurrence of such words. A statistical table can be built, for the probability of findings such words. Some investment strategy can then be decided, assuming the probability to vary weakly with time.

In Ausloos and Bronlet (2006), for example, two portfolios were invented, based on stocks in the $DJIA - 30$ and the $NASDAQ - 100$. After some time, two strategies with different weights for the shares in the portfolio at buying or selling time were imagined. For the next few years, the yearly expected return, variance, Sharpe ratio and β were calculated in each case; see Appendix 4 for the relevant definitions and comments. The best returns and weakest risks mainly depend on the chosen word length inducing the strategy. Even though some risk values could be large, returns were sometimes very high (Ausloos & Bronlet, 2006). Note that the investment time included exogenous and endogenous events. Much variability is clearly possible, depending on investor's choices.

One disadvantage of the Zipf method is that it does not easily distinguish persistent and antipersistent sequences.

Another method, a 'detrended fluctuation analysis method' (DFA) (Peng *et al.*, 1994) can also be used to build investment strategies. It is described here when discussing econophysics work on the foreign currency exchange markets.

C. Foreign Exchange Markets

The technical details of DFA are given in Appendix 5. Let it be known that the technique originated in Stanley group (Peng *et al.*, 1994) for sorting out DNA coding and non-coding regions. First note that, like for crashes, the (daily and weekly seasonal) volatility is of interest in the foreign exchange market (Dacorogna, Muller, Nagler, Olsen & Pictet, 1993). In particular, it was shown (Vandewalle & Ausloos, 1997) that the time dependent signal for foreign exchange currency rates does *not* obey the Brownian motion rule (coin tossing), but is rather a *fractional* Brownian motion (Addison, 1997; West & Deering, 1995). The 'diffusion' of the particle (= share price or index value) does not evolve like the square root of time, $t^{1/2}$, for large t, but has an exponent quite different from 0.5. Coherent and random sequences can be clearly seen in the financial fluctuations (Vandewalle & Ausloos, 1997, 1998d, 1998e). By the way, the analysis is robust against various *a priori* trends (Hu, Ivanov, Chen, Carpena & Stanley, 2001; Vandewalle & Ausloos, 1998b).

As an application, mention the evolution of foreign exchange currency rates, both for emerging markets and well-controlled ones. The exchange rates seem to belong to different 'categories' depending on the involved currencies: strictly regulated European, hard dollar zone, emerging markets, etc. (Ivanova & Ausloos, 1999b, 2002). The parameters are similar to those known in turbulence (Bergé *et al.*, 1987).

A multi-affine fractal-like analysis (Arnéodo, Argoul, Bacry, Elezgaray & Muzy, 1995), which however for lack of space will not be discussed further here, has also been implemented; see also Ausloos (2006) and Appendix 5.

Thus, not only long-term correlations exist and can be implemented for predictability, but some systematics of the short-range correlation functions through low order $i - variability$ diagrams (Babloyantz & Maurer, 1996) for short-range correlation evidence in, for example, exchange rate (and Gold price) are also observed (Ivanova & Ausloos, 1999a).

Very interestingly the features, like the characteristic exponent describing the evolution, sometimes present remarkable drastic turns. The features can be daringly associated to *economic* events following *political* events or some panic storm spreading over financial markets. It turns out that exogenous causes, for example, the Gulf War, the technological bubble explosion, ..., are, *a posteriori* (of course), well seen (Vandewalle & Ausloos, 1997). One can also distinguish an exponent behaviour quantitatively modified after some policy move for better control and in order to avoid panic or to heal a crisis. However, one can also observe/guess that the market agents thereafter likely search for the best subtle loopholes in the regulation, in order to avoid the most severe constraints, then slide off the policy main stream (Vandewalle & Ausloos, 1997) before new rules are decided upon.

D. Predictability

Thus, the most important question follows: knowing a distribution law of market fluctuations, may a winning strategy may be thought of and is predictability possible, on financial indices, on the currency exchange market, on the option market, ... ?

Yes. Among many cases, for example, several exchange rates and futures were studied in this respect, on a 16 year period. A robust (DFA) law for persistent and anti-persistent fluctuations was obtained. Thus, one was virtually able to increase an input capital by a factor of 40, but with less success for the price of gold, under 'perfect' conditions, that is, no tax no broker fee.

Usually, economists (i) do not admit such a type of predictability because they object about the lack of psychological features plus various theoretical matters, and (ii) estimate that the market is justified by real earnings of actors, in the long run. In the short run, economists, of course, admit fluctuations, based on (sudden, or not) events which create an atmosphere of optimism or pessimism that makes market actors systematically overestimate or underestimate future earnings. Economists claim that physical models cannot predict such surprises that suddenly change the optimism/pessimism in the market. They are right, except that physicists do not have to make such models. No physicist can predict an electrical power failure in the laboratory. But if a source of heat is removed, it is known that water may freeze when the temperature drops below 0°C—that is well known to physicists.

E. Price and Asset Distribution

The word 'price' has been mentioned several times here earlier. One more comment is in order, to show a connection

from basic physics to finance through econophysics, on this matter.

Many recent observations have indicated that the traditional *equilibrium market hypothesis* (*EMH*; also known as Effcient Market Hypothesis) is unrealistic. For example, long-term memory effects in stock market prices (Lo, 1991) and volumes (Lobato & Velasco, 2000) imply a non-Gaussian distribution of these. In fact, a price evolution along the EMH is analogous to a Boltzmann equation in physics, thus having some bad properties of mean-field approximations like a Gaussian distribution of price fluctuations (Ausloos, 2000), since such distributions and related ones have 'fat tails' (Lo, 1991; Lobato & Velasco, 2000; Mandelbrot, 1963, 1967). This *kinetic theory for prices* can be simply derived, considering in a first approach that market actors have all identical relaxation times, and solved within a Chapman-Enskog like formalism. In so doing, an equation of state is obtained linking a *pressure,* a *temperature*, that is, the inverse of the relaxation time, and *a volume*, the price (*being taken as the order parameter*) of a stock. This may lead to further studies on relations between intensive and extensive variables (Stanley, 1971) with applications as in 'less simple' technical analysis schemes (Ausloos & Ivanova, 2002, 2004).

A Boltzmann-type master equation for the problem of asset exchange and an analytic solution have been derived in Chatterjee and Chakrabarti (2005). It was shown also by Chatterjee, Chakrabarti and Manna (CCM) that kinetic models of money exchange with the added feature of 'savings' can nevertheless produce self-organization and a heavy-tailed money (asset) distribution for traders with distributed savings (Chatterjee, Chakrabarti & Manna, 2004). In Chatterjee, Chakrabarti and Manna (2004) a 'Pareto region' is found in the probability distribution, with exponent value $\simeq 1$. Previously, Chakraborti and Chakrabarti (2000) had numerically studied a random exchange model, which conserves money, and in which each agent saves a fixed fraction of their instantaneous money. This model (Chakraborti & Chakrabarti, 2000) was later reanalyzed by Das and Yarlagadda (2003) using a Boltzmann transport type formulation.

Instead of savings one can investigate taxes. A very simple model of a closed marked first with tax-free exchange of goods and wealth has been studied in Ausloos and Pekalski (2007): the amount of goods and the accepted price, as well as the trade decisions have no relation to trends, previous activity, anticipation, savings, etc. Also partners for trading are chosen randomly among all agents. Such a free market stabilizes itself 'rapidly', the average

price, number of transactions, number of goods sold and money paid for them, become asymptotically fixed in time. In some sense there is an intrinsically generated (self-organized) utility function. The distribution of money and goods shows a stratification of the society. (This was further confirmed in a study in which there is competition between size-dependent peer agents (Caram, Caiafa, Proto & Ausloos, 2010).) There seems to be a trend away from Pareto's values of 1.5 to slightly higher values. It can be wondered if this represents the impact of socialism policy since a high value 'fat tail' is associated with greater redistribution of money, as found by Ausloos and Pekalski (2007), whereas a low value of the Pareto exponent suggests that the rich really are rich and the poor are poor. When part of the money disappears from the market under the form of taxes, at each transaction, characteristics are quite similar to the no tax case, though there is a difference in the distribution of wealth, that is, the poor gets poorer and the rich gets richer.

Macro-econophysics Aspects

Rather more recently econophysics has been enlarged to studies of *macroeconomy*. It arose in the case of Ausloos, Clippe and Pekalski (2003) when searching to confirm or infirm, within econophysics modelling, political statements, like 'The North must help the South! It's good for the North economy.' However, this is hard to swallow by workers who see their work being delocalized. When will the growth part of the business cycle come back?

The ACP model (Ausloos *et al.*, 2003, 2004) has many ingredients: different geographic regions, initial concentration(s), economic field time sequence(s), selection pressure, diffusion process rule(s), enterprise-enterprise interaction(s), business plan(s), number of regions, enterprise evolution law(s) and economic policy time delay implementation, all presupposed to be known for the Monte-Carlo simulation. It is found that the model even in its simplest forms can lead to a large variety of situations, including: stationary solutions and cycles, but also chaotic behaviour.

This complexity led to simplify the macro-economic question to read: *is there any economy globalization?* The globalization process (Encyclopedia Britannica) can be discussed from a political (Verdirer & Breen, 2001), cultural (Robertson, 2000), scientific cooperation (Beaver, 2001) or economy (O'Rourke & Williamson, 1999) point of view. The problem has become so fashionable nowadays

that even so-called antiglobalization movements and meetings are organized (Bhagwati, 2002; Fleck, 2004). In Miskiewicz and Ausloos (2006a, 2008), the definition was restricted such that one could study the case of the world economy in a quantitative aspect, that is,

- A globalization process in economy is understood as the increase of similarities in development patterns.

The most important questions which arises thereafter this definition pertains to how to measure *similarities in patterns* and which economy parameters have to be taken into consideration for doing so. Much debate exists—many admit that the different approaches might intrinsically imply different (types of) conclusions.

In Miskiewicz and Ausloos (2006a, 2008), 'similarities' were searched for by measuring *distances* between the gross domestic product (GDP) time series. The GDP seems to be the most representative parameter describing the status of an economy, since, *mutatis mutandis*, it is defined for all countries. Data was taken from *http://www.ggdc.net*, 2006. *Differently defined 'distances'* between pairs of GDP series were considered, either based on the linear cross-correlation coefficient, that is, a *statistical* distance, or on a measure of the disorder level in the GDP evolution, that is, an *entropy* or information distance. It was found that the time averaging of the distances over finite size time windows is fundamental. It implies considerations, on policy effects, due to time delay in implementing policies. Nevertheless, some country hierarchy is obtained. Network structures can be constructed based on the hierarchy of distances. It was shown that the mean distance between the most developed countries on several networks decreased in time.

By the way, it was shown that the entropy distance measure is more suitable in detecting a globalization process than the usual statistical (correlation based) measure. Note that other macroeconomic indices, similarly studied in Miskiewicz and Ausloos (2010), confirm the findings. Moreover, the results indicated that the EUR introduction and the Maastricht agreement constraints induced the start of a deglobalization between European countries!

See also, but using a different measurement technique (Redelico, Proto & Ausloos, 2009), a related study on hierarchies and structures in the GDP per capita fluctuation in Latin American countries.

In the same line of questioning, it might be audacious to consider a country hierarchy when describing them through network schemes based on some ranking through macroeconomic indices. However, not only in doing so one obtains a way to elaborate on structured clusters, but also connect to Hamiltonian mechanics. More considerations can be found in Gligor and Ausloos (2007, 2008a, 2008b).

Perspectives and Prospects

Scientific progress is expected to depend on the combination of experimental and theoretical investigations that provide deep understanding of phenomena, with an absolute need for a high reliability of results. Moreover, simulation experiments can nowadays be part of theoretical or experimental advances; see the case of 'sociophysics' in Stauffer (2003a, 2003b). The simple ideas proposed here above on thermodynamic and sometimes geometric, phase transitions lead to financial markets studies and understandings, including sociological aspects. All that seems to be derived from the study of a dynamical state propagation in a random medium—lattices or networks. The fundamental ideas refer to correlations in fluctuations (Fisher, 1974), invasion percolation (Stauer & Aharony, 1992), and self-organized criticality (Bak *et al.*, 1987, 1988).

Many, many, other topics could be presented. For lack of space, let us point to a very (very) small set, with one reference only, for the interested reader to start from:

- increasing returns and economic geography; see Krugman (1990),
- modelling and pricing derivatives, even weather; see Alaton *et al.* (2002),
- discussing inequalities of wealth distribution, with the Gini coefficient; see Pianegonda and Iglesias (2004),
- assessing information flow time delay in the formation of economic cycles; see Miskiewicz and Ausloos (2006b),
- controllling shareholding networks through cross and joint ownership; see Rotundo and D'Arcangelis (2010),
- finances in sport; see Torgler (2004),
- price auctions, in e.g., art or other cultural matters; see Reddy and Dass (2006).

It is time to comment on 'methods' and 'models'—in fact adding to the remarks at the end of the 'Introduction' section.

- Methods

Tools of time series analysis, varying the scale of resolution, are very useful to characterize important features of complex systems. It is strongly recommended to Fourier transform whatever signal is investigated (Ausloos, 2001, 2002), in order to sort out the most important periodicities in the system: yearly, seasonal, weekly, daily, ...or others. This leads also to measure a parameter, the phase, which may depend on for example, different fiscal year terms.

Beside the techniques not mentioned so far, let the Recurrence Plot Analysis be pointed out. It has served to study critical regimes, like crashes (Fabretti & Ausloos, 2005, 2006; Guhathakurta *et al.*, 2010), among other features. The Theil, entropy-like mapping, of a time series (Miskiewicz, 2008) is also very interesting, in particular when searching whether Benford law (Benford, 1938; Newcomb, 1881) is valid (Clippe & Ausloos, 2012).

- Modelling

Let it be re-emphasized that several scientific issues can be found: for example, (i) creating models based on financial insights and mathematical principles, (ii) calibrating models based on market information and (iii) simulating models using specific algorithms.

From the statistical analysis of 'size-frequency distribution', an attachment process is often seen (through the 'fat tails' (Mandelbrot, 1963, 1967)) as the primary cause of the distribution (of returns, e.g.) evolution (Cont & Bouchaud, 2000). Such constraint aspects must be introduced in network descriptions of financial and economic matters. However, the modern description of human societies through networks is often lacking other mandatory ingredients, that is, the non scalar nature of the nodes and the non binary aspects of nodes and links, though for the latter this is already often taken into account, including directions,—but this quite complicates the algebra (Rotundo & Ausloos, 2013).

Coming back to roughly the two main themes (which might be thought as approximations) in micro-economphysics: there are based on the supposedly different goals and strategies of 'players' (in other words 'traders' or more generally 'agents'): chartists or fundamentalists, in so calling, remaining within the distinction made by micro-economy theories.

One has to consider competition like processes. Future trends in econophysics studies should connect better to reality in tying such features of agents and society. Indeed, changes in economic behaviour can occur either due to 'heterogeneous agent interaction' processes or due to 'external field' constraints—or both.

Beside the basic physics models recalled in the first section, and for which there are numerous references, let the recent fluctuating mass Brownian particle (Ausloos & Lambiotte, 2006) be understood as a physical analogy to a 'price.volume' variable (Ausloos & Ivanova, 2002, 2004),—what is often in fact the real constraint of investors.

Conclusions

One should re-emphasize that the very relevant item, which much pleases 'fractalists' and statistical physicists is the discovery (after several attempts) that the markets are not really 'efficient', but is 'scaling' (Mantegna & Stanley, 1995)—the (truncated) Levy distribution was rather a universal one in order to describe fluctuation correlations and evolving like order parameters in physics (Bouchaud & Cont, 1998). Bachelier analysis and the Gaussian hypothesis are not true (Peters, 1994). The Black-Scholes hypotheses are incorrect (Bouchaud & Sornette, 1994). Discrete Scale Invariance (Sornette, 1998) has to be inserted from the start in network models.

Economists scorn physics models because the former ones claim to have also endeavoured the prediction of short term fluctuations by examining past fluctuations (chart analysis) or approximating stock prices by wave structures (Elliott waves). These models, obviously, can only be a source of profits if they turn out to be a self-fulfilling prophecy. This can happen if many market actors believe in such theories. For example indeed, it has been shown that the mobile average technique is wholly unrealistic for predictability purposes (Vandewalle & Ausloos, 1998a). Therefore, economists are sending warnings about their own techniques, though not seeing differences with respect to physicist approaches. As a famous 'non-econophysicist' said once: 'If "it" occurs once and I can explain "it", I can explain "it" many times and "it" can occur many times.'

The need for all that can be criticized by physicists, mathematicians and economists, from first principles or on mathematical grounds. The econophysics approach should be taken with caution, indeed. The robustness and soundness of models are fundamental questions. Models should be predictive and should be tested. Zhang (1998) claims that there are too many directions of investigations. This could deserve the research. I believe that they are too many topics available at this time for the scientists involved as well. However, some self-organized framework might emerge. I am on this point rather optimistic (Ausloos, 1998).

Appendix 1. Log-Periodicity and Divergence

In two independent works (Feigenbaum & Freund, 1996; Sornette *et al.*, 1996), it was indicated that most economic indices, $y(t)$, follows a power law with a complex exponent, $m = m' + i\, m''$, such that

$$y(t) = A + B\left(\frac{t_c - t}{t_c}\right)^{-m'}\left[1 + C\left(\omega\ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad (A.1)$$

for $t < t_c$, where t_c is the crash-time or rupture point, the other symbols being parameters. A divergence occurs at $t = t_c$. This law generalizes the scaleless situation at thermodynamic critical points, so-called second order phase transitions (Fisher, 1974; Stanley, 1971), to cases in which discrete scale invariance exists (Anifrani *et al.*, 1995; Sornette, 1998). Superposed to the divergence, a series of oscillations could be observed,—as in the ionic content of water sources before some earthquakes (Johansen *et al.*, 1996). Interestingly, the period of these oscillations converges to the rupture point t_c. The modellization leads to a seven-parameter function for fitting the data.

Various values of m' were reported ranging from 0.53 to 0.06 for various indices and events (upsurges and crashes) (Feigenbaum & Freund, 1996), while m' ranges from 0.7 to 0.33 in (Sornette *et al.*, 1996). This lack of 'universality' suggests a strong error bar effects, due to many possible causes, thereby suggesting some simplification of the function as

$$y(t) = A + B\ln\left(\frac{t_c - t}{t_c}\right)\left[1 + C\left(\omega\ln\left(\frac{t_c - t}{t_c}\right) + \phi\right)\right] \quad (A.2)$$

for $t < t_c$. Moreover, as mentioned in the main text, this functional form reproduces known intriguing phenomena in physics, several of them allowing to mimic financial phenomena by analogy.

Whatever the functional form of the divergence, the log-periodic structure has been found in many studies. Beside those in the main text, let us mention (Drożdż *et al.*, 1999; Johansen *et al.*, 2000; Vandewalle *et al.*, 1999).

Appendix 2. Crash Predictability Technique

Thus, one can test such a (six-parameter) function) on financial indices, i.e., on one hand there is a three-parameter power law divergence (fixing $m = 0$), and on the other hand there is a log-periodic function, see Appendix 1, describing the oscillations of the financial index before the crash.

In so doing, one has monitored many indices after 1987. In 1997, the stock market numerical conditions astonishingly looked like the pre-crash period of 1987. Thus, in view of the huge similarity between the behaviours in 1987 and in 1997, and admitting

some universality of behaviour, one could come up with accurately predicting the October 27, 1997 crash (Dupuis, 1997a, 1997b; Legrand, 1997a, 1997b; Vandewalle & Ausloos, 1998c).

Practically, one can dissociate the fit to the divergence from the fit to the oscillations, whence having 2 three-parameter fits. One obtains *two*, usually different, t_c values, i.e., $t_c^{(d)}$ and $t_c^{(o)}$. Note that these values of t_c evolve with time during the signal analysis. If they converge to a single t_c, one can predict the theoretical crash time. The distance $t_c^{(d)} - t_c^{(o)}$ may also evolve smoothly—sometimes indicating a 'near-to-crash' or a 'missed crash'. A 'crash risk' notion can be invented.

Appendix 3. Zipf Method

The Zipf method (Zipf, 1949) consists in counting the frequency of 'something' in view of some ranking. In the original case, it was the number of words in a text. Zipf (1949) observed that a large number of such distributions, N_r can be approximated by a simple *scaling (power) law* $N_r = N_1/r$, where r is the ranking parameter, with $N_r \geq N_{r+1}$, (and obviously $r < r + 1$).

More generally, one can translate a time series into a text by choosing an appropriate alphabet to mimic the signal variations. For example, in finance, one can analyze in such a way, the number of fluctuations of a given size in a signal. One can choose a two letter alphabet (*u, d*) for *up* and *down* fluctuations; or a 3-letter alphabet (*u, s, d*) where *s* corresponds to *small* (as chosen by the analyst) fluctuations; or a five letter alphabet (*u, p, s, n, d*), etc. One counts the 'words', overlapping or not, either with equal 'length', or not. One ranks next the 'events' according to their frequency, in frequency decreasing order; the most frequent getting the rank $r = 1$, etc. The *rank—frequency* relationship, i.e., the frequency f of the occurrence of an 'event' relative to its rank r, usually reads like an inverse power law, $f \sim r^{-\zeta}$.

Note that one can also ask (Pareto, 1896) how many times one finds an 'event' greater than some size x, i.e., the 'size-frequency' relationship. Pareto's found out that the cumulative distribution function of such events, i.e., the number of events larger than F, (also) follows an inverse power of f, i.e., $P[f > F] \sim f^\lambda$ (Pareto, 1896). Theoretical work leads to $(1/\lambda) + \zeta = 2$. Recall that Pareto discovered that income distribution does not behave in a Gaussian way, but exhibits 'heavy tails'.

Appendix 4. Technical Definitions for Risk Evaluation

Call the relevant financial index, so called market (*M*) and portofolio (*P*) variance σ_M^2 and σ_P^2, respectively. The positive square root is called the standard deviation. Let $E(r_P)$ be the yearly returns.

The Sharpe ratio *SR*, given by $SR = E(r_P)/\sigma_P$, is considered to measure the portfolio *performance* (Sharpe, 1964).

The β is given by $cov(r_P, r_M)/\sigma_M^2$ where the P covariance $cov(r_P, r_M)$ is measured with respect to the relevant financial

index, i.e., $cov(r_P, r_M) = E(r_P, r_M) - E(r_P) E(r_M)$. The β is considered to measure the portfolio *risk*.

Appendix 5. Detrended Fluctuation Analysis (DFA)

The Detrended Fluctuation Analysis (DFA) technique was introduced in order to investigate long-range power-law correlations along DNA sequences Mandelbrot, 1967; Osborne, 1959). The method consists in dividing the whole discrete data sequence $X = \{x_i, i = 1, ..., N\}$ (in finance, X is a time series) of length N into non-overlapping boxes, each containing n points. It is suggested that the 'first' box contains the 'last' data points. In so doing, when N is incommensurate with a positive number k, one leaves out the 'oldest' signal values.

In the kth box, a 'new' signal is calculated: the wholly reduced-cumulative sum $Y_i^{(k)}$

$$Y_i^{(k)}(n) = \sum_{j=1}^{i}(x_j - \langle x \rangle_N) \quad i = kn + 1 - n, ..., kn \quad (A.3)$$

and $\langle x \rangle_N = (1/N)\sum_{j=1}^{N} x_j$.

In each box, one defines the local trend

$$z^{(k)}(n) = a^{(k)} n + b^{(k)} \quad (A.4)$$

as the ordinate of a linear least-square fit of the data points, in 'that' box. One should remark that a trend $z(n)$ different from a first-degree polynomial can also be used, such as the cubic trend [87].

Let

$$F_k^2(n) = \frac{1}{n}\sum_{i=kn+1-n}^{kn}\left(Y_i^{(k)}(n) - z^{(k)}(n)\right)^2, \quad (A.5)$$

The detrended fluctuation function is then calculated by averaging $F_k^2(n)$ over all the equal size intervals $\left(k = 1, 2, ..., \left(\dfrac{N}{n} - 1\right)\right)$. This gives a function depending on the box size n: $< F_k^2(n) >$. The calculation is repeated for all possible different box sizes n.

If the X data values are randomly uncorrelated variables or short range correlated variables, the behaviour of $\sqrt{\langle F_k^2(n) \rangle}$ is expected to be a power law

$$f(n) = \sqrt{\langle F_k^2(n) \rangle} \sim n^{\alpha}, \quad (A.6)$$

where α is in fact nothing else that the so-called Hurst exponent H for fractional Brownian motion (Addison, 1997; West & Deering, 1995). It can be useful to recall (Addison, 1997; West & Deering, 1995) that the power spectrum of such stochastic signals is characterized by a power law with an exponent $\beta = 2\alpha - 1$, which itself can be related to the fractal dimension of the signal.

The cases $\alpha > 1/2$ and $\alpha < 1/2$ should be physically distinguished. For $\alpha > 1/2$ there is so called persistence, i.e., $C > 0$. The $\alpha = 0$ situation corresponds to the so-called white noise. For $\alpha < 1/2$ the signal is said to be antipersistent, thus apparently very 'rough'.

For complementing Appendix 5, the autocorrelation function $C(i, t)$ of the signal $\{X_i\}_{i=1}^{N}$ is defined as

$$C(i, t) = \frac{\langle (X_t - \mu_t) \cdot (X_{i+t} - \mu_{i+t}) \rangle}{\sigma_t \cdot \sigma_{i+t}} = 2^{2\alpha - 1} - 1, \quad (A.7)$$

where σ_t is the standard deviation and μ_t is the mean value at the moment t.

The cases $\alpha > 1/2$ and $\alpha < 1/2$ should be physically distinguished For $\alpha > 1/2$ there is so called persistence, that is, $C > 0$. The $\alpha = 0$ situation corresponds to the so-called white noise. For $\alpha < 1/2$ the signal is said to be antipersistent, thus apparently very 'rough'.

References

Addison, P. S. (1997). *Fractals and chaos*. Bristol: Institute of Physics.

Alaton, P., Djehiche, B., & Stillberger, D. (2002). On modelling and pricing weather derivatives. *Applied Mathematical Finance, 9*, 1–20.

Anifrani, J. C., Le Floc'h, C., Sornette, D., & Souillard, B. (1995). Universal log-periodic correction to renormalization group scaling for rupture stress prediction from acoustic emissions. *J. Physique I (France), 5*, 631–638.

Arnéodo, A., Argoul, F., Bacry, E., Elezgaray, J., & Muzy, J. F. (1995). *Ondelettes, multifractales et turbulences*. Diderot Editeur, Arts et Sciences.

Asada, T. & Ouchi, M. (2013). Fiscal and monetary stabilization policies in a Keynesian model of endogenous growth cycle with public debt accumulation, Oral presentation at NED 2013.

Ausloos, M. (1998). The money games physicists play. *Europhys. News, 29*, 70–73.

Ausloos, M. (2000). Gas-kinetic theory and Boltzmann equation of share price within an equilibrium market hypothesis and ad hoc strategy. *Physica A, 284*, 385–392.

Ausloos, M. (2001). Financial time series and statistical mechanics. In K. H. Hoffmann & M. Schreiber (Eds), *Computational statistical physics. From billiards to Monte Carlo* (pp. 153–168). Berlin: Springer-Verlag.

Ausloos, M. (2002). Empirical analysis of time series. In W. Klonowski (Ed.), *From quanta to societies* (pp. 88–106). Lengerich: Pabst.

Ausloos, M. (2006). Econophysics of stock and foreign currency exchange markets. In B. K. Chakrabarti, A. Chakraborti & A. Chatterjee (Eds), *Econophysics and sociophysics* (pp. 249–278). Weinheim: Wiley-VCH, Weinheim.

Ausloos, M. (2012). Econophysics of a religious cult: The Antoinists in Belgium [1920–2000]. *Physica A, 391*, 3190–3197.

Ausloos, M., & Bronlet, Ph. (2003). Strategy for investments from Zipf law(s). *Physica A, 324*, 30–37.

Ausloos, M., & Bronlet, Ph. (2006). Risk portfolio management under Zipf analysis based strategies. In H. Takayasu (Ed.), *Practical fruits of econophysics* (pp. 257–261). Tokyo: Springer.

Ausloos, M., Clippe, P., & Pekalski, A. (2003). Simple model for the dynamics of correlations in the evolution of economic entities under varying economic conditions. *Physica A, 324*(1), 330–337.

Ausloos, M., Clippe, P., & Pekalski, A. (2004). Model of macroeconomic evolution in stable regionally dependent economic fields. *Physica A, 337*(1), 269–287.

Ausloos, M., & Ivanova, K. (2002). Mechanistic approach to generalized technical analysis of share prices and stock market indices. *Eur. Phys. J B, 27*, 177–187.

Ausloos, M., & Ivanova, K. (2004). Generalized technical analysis. Effects of transaction volume and risk. In H. Takayasu (Ed.), *The applications of econophysics* (pp. 117–124). Berlin: Springer Verlag.

Ausloos, M., Ivanova, K., & Vandewalle, N. (2002). Crashes: Symptoms, diagnoses and remedies. In H. Takayasu (Ed.), *Empirical sciences of financial fluctuations. The advent of econophysics* (pp. 62–76). Berlin: Springer-Verlag.

Ausloos, M., & Lambiotte, R. (2006). Brownian particle having a fluctuating mass. *Phys. Rev. E, 73*, 11105.

Ausloos, M., Leburton, J. P., & Clippe, P. (1980). Effect of magnetic domains on the electrical resistivity of a ferromagnet just below the critical temperature. *Solid State Commun, 33*, 75–77.

Ausloos, M., Miškiewicz, J., & Sanglier, M. (2004). The durations of recession and prosperity: Does their distribution follow a power or an exponential law? *Physica A, 339*, 548–558.

Ausloos, M., & Pekalski, A. (2007). Model of wealth and goods dynamics in a closed market. *Physica A, 373*, 560–568.

Babloyantz, A., & Maurer, P. (1996). A graphical representation of local correlations in time series: Assessment of cardiac dynamics. *Phys. Lett. A, 221*, 43–55.

Bachelier, L. (1900). Théorie de la spéculation. *Ann. Sci. Ecole Norm. Sup. 3*, 21–86. [English translation in P. Cootner (Ed.), *The random character of stock prices*. MIT Press, 1964, reprinted Risk Books, London, 2000.]

Bak, P. (1996). *How nature works: The science of self-organized criticality*. New York: Copernicus.

Bak, P., Tang, C., & Wiesenfeld, K. (1987). Self-organized criticality: An explanation of 1/f noise. *Phys. Rev. Lett., 59*, 381–384.

Bak, P., Tang, C., & Wiesenfeld, K. (1988). Self-organized criticality. *Phys. Rev. A, 38*, 364–374.

Barabási, A.-L., & Vicsek, T. (1991). Multifractality of self-affine fractals. *Phys. Rev. A, 178*, 2730–2733.

Bates, D. S. (1991). The crash of '87: Was it expected? The evidence from options markets. *J. Finance, 46*, 1009–1044.

Beaver, D. deB. (2001). Reflections on scientific collaborations (and its study): Past, present and prospective. *Scientometrics, 52*, 365–377.

Benford, F. (1938). The law of anomalous numbers. *Proc. Amer. Philos. Soc., 78*, 551–572.

Bergé, P., Pomeau, Y., & Vidal, C. (1987). Order within chaos: Towards a deterministic approach to turbulence. New York: Wiley.

Bhagwati, J. N. (2002). Coping with antiglobalization: A trilogy of discontents. *Foreign Affairs, 81*, 2–7.

Black, F., & Scholes, M. (1973). The pricing of options and corporate liabilities. *J. Political Econom, 81*, 637–654.

Bouchaud, J.-P., & Cont, R. (1998). A Langevin approach to stock market fluctuations and crashes. *Eur. Phys. J. B, 6*, 543–550.

Bouchaud, J. P., & Sornette, D. (1994). The Black-Scholes option pricing problem in mathematical finance: Generalization and extensions for a large class of stochastic processes. *J. Phys I France, 4*, 863–881.

Brisbois, E., Boveroux, Ph., Ausloos, M., & Vandewalle, N. (2000). On the occurrence of financial crashes. *Int. J. Theor. Appl. Fin., 3*, 423–424.

Bronlet, Ph., & Ausloos, M. (2003). Generalized (m,k)-Zipf law for fractional Brownian motion-like time series with or without effect of an additional linear trend. *Int. J. Mod. Phys. C., 14*, 351–365.

Caram, L. F., Caiafa, C. F., Proto, A. N., & Ausloos, M. (2010). Dynamic peer-to-peer competition. *Physica A, 389*, 2628–2636.

Chakraborti, A., & Chakrabarti, B. K. (2000). Statistical mechanics of money: How saving propensity affects its distribution. *Eur. Phys. J. B, 17*, 167–170.

Chatterjee, A., Chakrabarti, B. K., & Manna, S. S. (2004). Pareto law in a kinetic model of market with random saving propensity. *Physica A, 335*, 155–163.

Chatterjee, A., Chakrabarti, B. K., & Stinchcombe, R. B. (2005). Master equation for a kinetic model of a trading market and its analytic solution. *Phys. Rev. E, 72*, 026126.

Ciecka , J. E. (2008). Edmond Halley's life table and its uses. *Journal of Legal Economics, 15*(1), 65–74.

Clippe, P., & Ausloos, M. (2012). Benford's law and Theil transform of financial data. *Physica A, 391*, 6556–6567.

Cohen, I. B., Whitman, A., & Budenz, J. (1999). *Translation of The Principia: Mathematical Principles of Natural Philosophy by Isaac Newton*. University of California Press.

Cont, R. (2007). Volatility clustering in financial markets: Empirical facts and agent-based models. In A. Kirman & G. Teyssiere (Eds), *Long memory in economics* (pp. 289–309). Springer.

Cont, R., & Bouchaud, J.-P. (2000). Herd behaviour and aggregate fluctuations in financial markets. *Macroeconomics Dynamics, 4*, 170–196.

Dacorogna, M. M., Muller, U. A., Nagler, R. J., Olsen, R. B., & Pictet, O. V. (1993). A geographical model for the daily and weekly seasonal volatility in the foreign exchange market. *Journal of International Money & Finance, 12*, 413–438.

Das, A., & Yarlagadda, S. (2003). Analytic treatment of a trading market model. *Physica Scripta*, T106, 39.

Della Vigna, S., List, J. A., & Malmendier, U. (2012). Testing for altruism and social pressure in charitable giving. *Quarter. J. Econ., 127*, 1–56.

Drożdż, S., Ruf, F., Speth, J., & Wojcik, M. (1999). Imprints of log-periodic self-similarity in the stock market. *Eur. Phys. J. B, 10*, 589–593.

Dupuis, H. (1997a). *Trends-Tendances, 38*, 26–27 (in French).

Dupuis, H. (1997b). *Trends-Tendances, 44*, 11 (in French).

Encyclopedia Britannica. http://www.britannica.com/eb/article-9344667/globalization

Fabretti, A., & Ausloos, M. (2005). Recurrence plot and recurrence quantification analysis techniques for detecting a critical regime. Examples from financial market indices. *Int. J. Mod. Phys. C, 16*, 671–706.

Fabretti, A., & Ausloos, M. (2006). Recurrence analysis near the NASDAQ crash of April 2000. In H. Takayasu (Ed.), *Practical fruits of econophysics* (pp. 52–56). Tokyo: Springer.

Feigenbaum , J. A., & Freund, P. G. O. (1996). Discrete scale invariance in stock markets before crashes. *Int. J. Mod. Phys. B, 10*, 3737–3745.

Feigenbaum, M. J. (1980). The metric universal properties of period doubling bifurcations and the spectrum for a route to turbulence. *Ann. New York. Acad. Sci., 357*, 330–336.

Fisher, M. E. (1974). The renormalization group in the theory of critical phenomena. *Rev. Mod. Phys., 46*, 597–616; Err. *Rev. Mod. Phys., 47*, 543(1976).

Fleck, F. (2004, 21 January). Global economic outlook: Anti-globalization forces shift to pragmatic tactics. *International Herald Tribune*.

Gleick, J. (2004). *Isaac Newton*. UK: Harper Collins.

Gligor, M., & Ausloos, M. (2007). Cluster structure of EU-15 countries derived from the correlation matrix analysis of macroeconomic index fluctuations. *Eur. Phys. J B, 57*, 139–146.

Gligor, M., & Ausloos, M. (2008a). Convergence and cluster structures in EU area according to fluctuations in macroeconomic indices. *Journal of Economic Integration, 23*, 297–330.

Gligor, M., & Ausloos, M. (2008b). Clusters in weighted macroeconomic networks: The EU case. Introducing the overlapping index of GDP/capita fluctuation correlations. *Eur. Phys. J. B, 63*, 533–539.

Guhathakurta, K., Bhattacharya, B., & Chowdhury, A. R. (2010). Using recurrence plot analysis to distinguish between endogenous and exogenous stock market crashes. *Physica A, 389.9*, 1874–1882.

Hall, T., & Froderberg, A. (1970). *Carl Friedrich Gauss: A biography*. MIT Press Cambridge.

Halley, E. (1693). An estimate of the degrees of mortality of mankind, drawn from the curious tables of the births and funerals at the city of Breslaw, with an attempt to ascertain the price of annuities upon lives. *Philosophical Transactions, 17*, 596–610.

Haven, E. (2002). A discussion on embedding the Black-Scholes option pricing model in a quantum physics setting. *Physica A, 304*, 507–524.

Holton, G. A. (2004). Defining risk. *Financial Analysts Journal, 60*(6), 19–25.

Hu, K., Ivanov, P. Ch., Chen, Z., Carpena, P., & Stanley, H. E. (2001). Effect of trends on detrended fluctuation analysis. *Physical Review E, 64*, 011114.

Ivanova, K., & Ausloos, M. (1999a). Low order variability diagrams for short range correlation evidence in financial data: BGL-USD exchange rate, Dow-Jones Industrial Average, Gold ounce price. *Physica A, 265*, 279–286.

Ivanova, K., & Ausloos, M. (1999b). Low q-moment multifractal analysis of Gold price, Dow Jones Industrial Average and BGL-USD exchange rate. *Eur. Phys. J. B, 8*, 665–669; Err. 12, 613 (1999).

Ivanova, K., & Ausloos, M. (2002). False EUR exchange rates vs. DKK, CHF, JPY and USD. What is a strong currency? In H. Takayasu (Ed.), *Empirical sciences of financial fluctuations. The advent of econophysics* (pp. 51–60). Berlin: Springer-Verlag.

Johansen, A. (2002). Comment on Are financial crashes predictable? *Europhysics Letters, 60*, 809–810.

Johansen, A., Ledoit, O., & Sornette, D. (2000). Crashes as critical points. *International Journal of Theoretical and Applied Finance, 3*(2), 219–255.

Johansen, A., & Sornette, D. (1999). Financial 'anti-bubbles': Log-periodicity in gold and Nikkei collapses. *Int. J. Mod. Phys. C, 10*, 563–575.

Johansen, A., & Sornette, D. (2000). The NASDAQ crash of April 2000: Yet another example of log-periodicity in a speculative bubble ending in a crash. *Eur. J. Phys. B, 17*, 319–328.

Johansen, A., & Sornette, D. (2010). Shocks, crashes and bubbles in financial markets. *Brussels Economic Review (Cahiers économiques de Bruxelles), 53*, 201–253.

Johansen, A., Sornette, D., & Ledoit, O. (1999). Predicting financial crashes using discrete scale invariance. *J. Risk, 1*, 5–32.

Johansen, A., Sornette, D., Wakita, H., Tsunogai, U., Newman, W. I., & Saleur, H. (1996). Discrete scaling in earthquake precursory phenomena: Evidence in the Kobe earthquake, Japan. *J. Phys. I (France), 6*, 1391–1402.

Kosterlitz, J. M., & Thouless, D. J. (1972). Long range order and metastability in two dimensional solids and superfluids (Application of dislocation theory). *J. Phys. C, 5*, L124–L126.

Kosterlitz, J. M., & Thouless, D. J. (1973). Ordering, metastability and phase transitions in two-dimensional systems. *J. Phys. C, 6*, 1181–1203.

Krugman, P. (1990). Increasing returns and economic geography. National Bureau of Economic Research.

Kwapień, J., & Drożdż, S. (2012). Physical approach to complex systems. *Physics Reports, 515*, 115–226.

Laloux, L., Potters, M., Cont, R., Aguilar, J. P., & Bouchaud, J. P. (1998). Are financial crashes predictable? *Europhys. Lett., 45*, 1–5.

Lambiotte, R., & Ausloos, M. (2005). Uncovering collective listening habits and music genres in bipartite networks. *Physical Review E, 72*, 066107.

Lambiotte, R., & Ausloos, M. (2006). Endo- vs. exo-genous shocks and relaxation rates in book and music 'sales'. *Physica A, 362*, 485–494.

Lauterbach, B., & Ben-Zion, U. (1993). Stock market crashes and the performance of circuit breakers: Empirical evidence. *J. Finance, 48*, 1909–1925.

Legrand, G. (1997a). *Cash, 38*, (in French).

Legrand, G. (1997b). *Cash, 44*, 3 (in French).

Lo, A. (1991). Long-term memory in stock market prices. *Econometrica, 59*, 1279–1313.

Lobato, I., & Velasco, C. (2000). Long memory in stock market trading volume. *Journal of Business and Economic Statistics, 18*, 410–427.

Lorenz, E. N. (1963). Deterministic nonperiodic flow. *J. Atmos. Sci., 20*, 130–141.

Lux, T., & Marchesi, M. (2000). Volatility clustering in financial markets: A microsimulation of interacting agents. *International Journal of Theoretical and Applied Finance, 3*(04), 675–702.

Mandelbrot, B. B. (1963). Certain speculative prices. *The Journal of Business, 45*(4), 542–543.

Mandelbrot, B. B. (1967). The variation of some other speculative prices. *J. Business, 36*, 349–413.

Mantegna, R. N., & Stanley, H. E. (1995). Scaling behaviour in the dynamics of an economic index. *Nature, 376*, 46–49.

Mantegna, R. N., & Stanley, H. E. (1999). *An introduction to econophysics: Correlations and complexity in finance*. Cambridge: Cambridge University Press.

Markowitz, H. (1959). *Portfolio selection: Efficient diversification of investments*. New York: John Wiley and Sons.

Markowitz, H. M. (1952). Portofolio selection. *J. Finance, 8*, 77–91.

Marshall, F. (1994). The role of the investment horizon in optimal portfolio sequencing an intuitive demonstration in discrete time. *Financial Review, 29*, 557–576.

Merton, R. C. (1973). The rational theory of option pricing. *Bell J. Econ. Manag. Sci., 4*, 141–183.

Merton, R. C. (1990). *Continuous-time finance*. Cambridge: Blackwell.

Miskiewicz, J. (2008). Globalization entropy unification through the Theil index. *Physica A, 387*, 6595–6604.

Miskiewicz, J., & Ausloos, M. (2006a). An attempt to observe economy globalization: The cross correlation distance evolution of the top 19 GDP's. *Int. J. Mod. Phys. C, 17*, 317–332.

Miskiewicz, J., & Ausloos, M. (2006b). Influence of information flow in the formation of economic cycles. In M. Ausloos & M. Dirickx (Eds), The logistic map and the route to chaos (pp. 223–238). Berlin: Springer.

Miskiewicz, J., & Ausloos, M. (2008). Correlation measure to detect time series distances, whence economy globalization. *Physica A, 387*, 6584–6594.

Miskiewicz, J., & Ausloos, M. (2010). Has the world economy reached its globalization limit? *Physica A, 389*, 797–806.

Newcomb, S. (1881). Note on the frequency of use of the different digits in natural numbers. *Amer. J. Math., 4*, 39–40.

Nicolis, G., & Prigogine, I. (1977). *Self-organization in nonequilibrium systems: From dissipative structures to order through fluctuations*. New York: Wiley.

O'Rourke, K. H., & Williamson, J. G. (1999). *Globalization and history. The evolution of a nineteenth-century Atlantic economy*. MIT Press.

Osborne, M. F. M. (1959). Brownian motion in the stock market. *Operations Research, 10*, 345–379.

Pareto, V. (1896). *Cours d'Economie Politique*. Geneva, Switzerland: Droz.

Peng, C.-K., Buldyrev, S. V., Havlin, S., Simmons, M., Stanley, H. E., & Goldberger, A. L. (1994). Mosaic organization of DNA nucleotides. *Phys. Rev. E, 49*, 1685–1689.

Peters, E. E. (1994). *Fractal market analysis*. New York: J. Wiley.

Pianegonda, S., & Iglesias, J. R. (2004). Inequalities of wealth distribution in a conservative economy. *Physica A, 342*, 193–199.

Reddy, S. K., & Dass, M. (2006). Modeling on-line art auction dynamics using functional data analysis. *Statistical Science, 21*(2), 179–193.

Redelico, F. O., Proto, A. N., & Ausloos, M. (2009). Hierarchical structures in the gross domestic product per capita fluctuation in Latin American countries. *Physica A, 388*, 3727–3535.

Robertson, R. (2000). *Globalization: Social theory and global culture*. London: SAGE Publications.

Roll, R. (1989). *The international crash of October 1987, Black Monday and the future of financial markets*. Irwin, Homewood, IL: Mid-America Institute for Public Policy Research Inc.

Ross, A., Garcia Cantù, & Ausloos, M. (2009). Organizational and dynamical aspects of a small network with two distinct communities: Neo creationists vs. evolution defenders. *Scientometrics, 80*, 457–472.

Rotundo, G., & Ausloos, M. (2013). Complex-valued information entropy measure for networks with directed links (digraphs). Application to citations by community agents with opposite opinions. *Eur. Phys. J. B, 86*: 169.

Rotundo, G., & D'Arcangelis, A. M. (2010). Ownership and control in shareholding networks. *J. Econ. Inter. Coord., 5*, 191–219.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance, 19*, 425–442.

Sornette, D. (1998). Discrete-scale invariance and complex dimensions. *Phys. Rep., 297*, 239–270.

Sornette, D. (2006). Endogenous versus exogenous origins of crises. In *Extreme events in nature and society* (pp. 95–119). Springer.

Sornette, D., & Helmstetter, A. (2003). Endogenous versus exogenous shocks in systems with memory. *Physica A, 318*, 577–591.

Sornette, D., Johansen, A., & Bouchaud, J. P. (1996). Stock market crashes, precursors and replicas. *J. Phys. I (France), 6*, 167–175.

Sousa, J. B., Amado, M. M., Pinto, R. P., Moreira, J. M., Braga, M. E., Ausloos, M., Leburton, J. P., Clippe, P., Van Hay, J. C., & Morin, P. (1979). Critical behaviour of transport coefficients at a structural ferromagnetic transition. *J. Phys., 40*, C5, 42–43.

Sousa, J. B., Amado, M. M., Pinto, R. P., Moreira, J. M., Braga, M. E., Ausloos, M., Leburton, J. P., Van Hay, J. C., Clippe, P., Vigneron, J. P., & Morin, P. (1980). Critical behaviour of transport coefficients at a structural ferromagnetic transition: I. Electrical resistivity of TbZn. *J. Phys. F, 10*, 933–945.

Stanley, H. E. (1971). *Phase transitions and critical phenomena*. Oxford: Oxford University Press.

Stauffer, D. (1999). Does econophysics make sense? Opinion for Fribourg Univ. Econophysics Forum electronic bulletin board. Retrieved from www.unifr.ch/econophysics

Stauffer, D. (2003a). Sociophysics—A review of recent Monte Carlo simulations. *Fractals, 11*, 313–318.

Stauffer, D. (2003b). Sociophysics simulations. *Computing in Science and Engineering, 5*, 71–75.

Stauffer, D., & Aharony, A. (1992). *Introduction to percolation theory*. London: Taylor & Francis.

Stauffer, D., & Pandey, R. B. (2000). Search for log-periodic oscillations in stock market simulations. *Int. J. Theor. Appl. Finance, 3*, 479–482.

The Conference Board and Groningen Growth and Development Centre, Total Economy Database. http://www.ggdc.net, 2006.

Torgler, B. (2004). The economics of the FIFA Football Worldcup. *KYKLOS, 57*, 287–300.

Vandewalle, N., & Ausloos, M. (1997). Coherent and random sequences in financial fluctuations. *Physica A, 246*, 454–459.

Vandewalle, N., & Ausloos, M. (1998a). Crossing of two mobile averages: An original method for measuring the roughness exponent. *Phys. Rev. E, 58*, 6832–6834.

Vandewalle, N., & Ausloos, M. (1998b). Extended detrended fluctuation analysis for financial data. *Int. J. Comput. Anticipat. Syst., 1*, 342–349.

Vandewalle, N., & Ausloos, M. (1998c). How the financial crash of October 1997 could have been predicted. *Eur. J. Phys. B, 4*, 139–141.

Vandewalle, N., & Ausloos, M. (1998d). Multi-affine analysis of typical currency exchange rates. *Eur. J. Phys. B, 4*, 257–261.

Vandewalle, N., & Ausloos, M. (1998e). Spareness and roughness of foreign exchange rates. *Int. J. Mod. Phys. C, 9*, 711–720.

Vandewalle, N., Ausloos, M., Boveroux, Ph., & Minguet, A. (1999). Visualizing the log-periodic pattern before crashes. *Eur. J. Phys. B, 9*, 355–359.

Vandewalle, N., Boveroux, Ph., Minguet, A., & Ausloos, M. (1998). The crash of October 1987 seen as a phase transition amplitude and universality. *Physica A, 255*, 201–210.

Verdirer, D., & Breen, R. (2001). Europeanization and globalization politics against markets in the European Union. *Comparative Political Studies, 34*, 227–262.

West, B. J., & Deering, W. D. (1995). *The lure of modern science: Fractal thinking*. World Scientific.

Westfall, R. (1994). *The life of Isaac Newton*. Cambridge: Cambridge University Press.

Zhang, Y. C. (1998). Evolving models of financial markets. *Europhys. News, 29*, 51.

Zipf, G. K. (1949). *Human behaviour and the principle of least effort: An introduction to human ecology*. Cambridge: Addison Wesley.