Article

Modelling Bullwhip Effect in a Closed Loop Supply Chain with ARMA Demand

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Abstract

The growing importance of environmental concerns and focus on recycling has encouraged our research efforts to study the bullwhip effect on Closed Loop Supply Chain (CLSC). This article attempts to measure bullwhip effect in a six echelon CLSC with recycling as a reprocessing option for materials like paper and plastics (perhaps the first time in CLSC literature). The proposed model considers the effect of two critical parameters of CLSC: quality of recyclable raw material (in terms of its yield) and degree of segregation at source. It is assumed that the echelons in CLSC employ an Order-Up-To (OUT) inventory policy with Minimum Mean Square Error (MMSE) forecasting scheme, and that the demand forecast is performed via a first order Auto Regressive Moving Average model (ARMA [1, 1]). The model developed is employed to investigate the impact of Auto Regressive (AR) and Moving Average (MA) parameter, yield of recycled material, degree of segregation at source and the lead-time on the bullwhip effect. The model enables the CLSC managers to anticipate the downstream demand. In order to avoid order-process instability, a careful selection of Auto Regressive (AR) and the MA parameters is advised. Sensitivity analysis on replenishment lead-times provides managerial insights for effective design of recycling—distribution system, with constant accumulated lead-time. Further, the analysis reveals that increased degree of segregation at the source reduces the bullwhip effect.

Keywords

Environmental concerns, recycling, closed loop supply chain, bullwhip effect, ARMA demand process

Introduction

Improved standard of living in the modern industrialized society has increased pressure on the ecosystem due to depletion of the natural resources and available landfills for end-of-life (EOL) waste. The resultant raw material shortage and/or legislation for environmental concerns require adoption of sustainable economic development strategies through 'product-take-back' policies. A typical Closed Loop Supply Chain (CLSC) attempts to efficiently integrate both the forward and the reverse logistics activities (like collection, inspection/separation, reprocessing and redistribution or disposal). While, forward logistics involves material flow from manufacturer to the end user, reverse logistics involves flows of used/waste/obsolete product returns from the end-user, for reprocessing (through recycling, cannibalizing, remanufacturing, refurbishing, repairing and/or reusing) the product's into a useable form (Thierry et al., 1995).

Bullwhip effect or variance amplification phenomenon refers to increase in demand variability as perceived by the upstream echelons in the supply chain to satisfy the demand at the retail outlet. Lee et al. (1997a, 1997b) studied the side-effects of variance amplification along with its major causes, and also suggested strategies for reducing the same. Many attempts have been made by researchers to measure bullwhip effect. The bullwhip ratio (σ_O^2/σ_D^2) proposed by Chen et al. (2000a) for two stage problem is one of the most popular measures of the bullwhip effect, where σ_D^2 denotes the variance of demand and σ_O^2 refers to the variance of order placed by a retailer on the wholesaler.

Bullwhip effect is an extensively studied phenomenon but only for the forward supply chain component of a CLSC. Number of researchers (Agrawal et al., 2009; Chen et al., 2000a, 2000b; Kim & Ryan, 2003; Lee et al., 1997a, 2000; Luong, 2007; Luong & Phien, 2007; Zhang, 2004) have quantified the bullwhip effect for a simple two-stage serial supply chain with the retailer facing Auto Regressive (AR) demand process of degree one or two (i.e., AR (1) and AR (2), respectively) and Order-Up-To (OUT) inventory policies. Table 1 highlights the key findings of the earlier mentioned statistical approaches.

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Table 1. Summary of Literature on Measuring Bullwhip Effect Statistically

Statistical Approach

Reference	Retailer's Demand Process	Forecasting Method Used	Findings/Suggestions
Lee et al. (1997a)	AR (1)	ES	Bullwhip effect is caused by the need to forecast. The bullwhip effect is smaller for a smoother demand forecasts.
Lee et al. (2000)	AR (1)	MMSE	Demand information sharing reduces bullwhip.
Chen et al. (2000a)	AR (1)	MA	Recognize the role of demand forecasts as a filter for the bullwhip effect.
Chen et al. (2000b)	AR (1)	ES	Reducing the lead-time and centralization of demand information can reduce the bullwhip effect.
Kim and Ryan (2003)	AR (1)	EWMA	Increase in order variance is enlarged by increasing the lead-time and reducing level of information sharing.
Zhang (2004)	AR (1)	Compares MMSE, MA, ES	MMSE method is optimal the best as because it leads to the lowest inventory cost.
Agrawal et al. (2009)	AR (1)	MMSE	Some part of bullwhip effect was observed to always remain even after sharing both inter- as well as intra-echelon information. The lead-time reduction is more beneficial in comparison to the sharing of information in terms of reduction in the bullwhip effect phenomenon.
Luong (2007)	AR (1)	MMSE	With increase in lead-time, the value of auto regressive coefficient at which the bullwhip effect reaches its maximum value increases and approaches one.
Luong and Phien (2007)	AR (2)	MMSE	Auto regressive coefficients and lead-time have a large impact on the bullwhip effect.

Notes: ES = Exponential Smoothing; EWMA = Exponential Weighted Moving Average; MMSE = Minimum Mean Square Error; MA = Moving Average; AR = Auto Regressive.

Control systems engineering approach has also been used (e.g., Dejonckheere et al., 2003, 2004; Disney et al., 2006; Duc et al., 2008; Gaalman & Disney, 2006, 2009 etc.) for measuring the bullwhip effect in the two stage serial supply chain under OUT inventory policies. Hosoda and Disney (2006) modelled and analyzed a three echelon supply chain with AR (1) demand for retail consumer. The study obtained exact analytical expressions for bullwhip ratio and net inventory variance at each echelon in supply chain. Their analysis reveals that bullwhip effect is influenced by the cumulative lead-time from customer end to the concerned local echelon.

The literature on the impact of demand variability on the variance amplification across the CLSC is very limited. Some studies using control engineering approach have developed simple dynamic model for a hybrid manufacturing/remanufacturing CLSC. Tang and Naim (2004) indicated that increased information transparency enhances the robustness of the hybrid system. Zhou et al. (2004), Zhou and Disney (2006) and Huang and Wang (2007) investigated the effect of remanufacturing lead-time and the return rate on the inventory variance and bullwhip effect produced by the ordering policy.

This article utilizes the concept of Hosoda and Disney (2006) for three echelon forward supply chain with AR (1) retailer demand process and extends it to the entire domain of CLSC combining the forward and reverse supply chain together and, examines on the bullwhip effect with enriched Auto Regressive Moving Average model (ARMA (1, 1)) demand process whereas Pati et al. (2010) also worked with AR (1) retails demand process in the above setup. The article attempts to model a more realistic CLSC by statistically investigating the bullwhip effect across a six echelon CLSC with recycling as the only feasible reprocessing option. The first three echelons are the members of forward supply chain and the rest are part of reverse supply chain. The bullwhip effect is expected to be different in the reverse supply chain part of the CLSC due to the movement of varying quality of each returned materials; of which some has to be segregated and disposed. In addition to incorporating reverse supply chain domain to the model by Hosoda and Disney (2006), the proposed model also includes the impact of the yield of the recyclable materials at the manufacturer stage.

The model assumes that the customer demand follows a stochastic first order Auto Regressive Moving Average process (ARMA[1, 1]). Gilbert (2005) suggested that OUT inventory policy a good ordering algorithm to achieve customer service, as well as, balance inventory/capacity investments. Hence, it is assumed that each of the CLSC participants adopt the OUT policy with Minimum Mean Square Error (MMSE) forecasting scheme. The demand for post consumption recyclable materials by the manufacturer depends upon:

- *The quality of the materials being collected during recycling*: Higher the yield (one of the quality indicators) of the recyclable materials at the manufacturer, higher is the demand for recycled material and consequently lower is the consumption of natural resources as raw materials.
- *The proportion of disposable waste moving between echelons:* The higher the segregation of non-recyclable materials at the initial stage, the lower is the demand in subsequent stages of product flow (as waste material flow gets reduced).

The model takes into consideration the impact of degree of segregation (in terms of Ψ, that is, proportion of recyclable materials in the initial collection), the quality of recyclable material input at manufacturer (in terms of yield [X] of recyclable materials) and lead-time between echelons on the bullwhip effect across the CLSC with recycling. This, article seeks to examine the following issues in the context of CLSC described earlier:

- Impact of providing an improved segregation system on the bullwhip effect.
- Effect of quality of recyclable materials on bullwhip effect.
- Impact of lead-time between entities on demand variability across the CLSC.
- Can the upstream partners of CLSC anticipate the demand pattern if customer demand process is ARMA (1, 1) and the entities use OUT replenishment policy with MMSE forecasting scheme?

Demand Model

Consider a retailer's single item multi-period inventory problem. The retailer orders and replenishes its stock from a wholesaler on a fixed time interval to fulfil customer demand. The lead-time between ordering and receiving of the goods is l_1; the excess inventory is assumed to be returned without cost and it is assumed that the retailer experiences an ARMA (1, 1) demand process. The formulation of the demand process is given (Box & Jenkins, 1994) as:

$$D_t = \rho D_{t-1} + \varepsilon_t - \theta \varepsilon_{t-1}, \text{ Where } |\rho| \le 1, |\theta| \le 1 \qquad (1)$$

Where, D_t is the demand during period t; ρ is Auto Regressive (AR) parameter, ε_t is a i.i.d. normally distributed random error with mean 0 and variance σ_ε^2; and θ is Moving Average (MA) parameter. The assumption of $|\rho| < 1$ assures that the demand process is covariance stationary and $|\theta| \le 1$ assures the condition of invertibility. Therefore, demand variance that retailer experiences due to the ARMA (1, 1) demand pattern from the customer (C) is given by:

$$\sigma_C^2 = \left(\frac{1+\theta^2-2\rho\theta}{1-\rho^2}\right)\sigma_\varepsilon^2 \qquad (2)$$

Measure of Bullwhip Effect in Recycling Closed Loop Supply Chain (CLSC)

The CLSC with recycling as the reprocessing option consists of six echelons (that is, retailer, wholesaler, manufacturer, supplier, segregator and the dealer as shown in Figure 1) and attempts to capture the real business practices followed in developing countries. The quantity of waste collected at customer level is not sufficient to permit direct transportation to manufacturer economically. The demand for a specific recycled product is made by the customer to the retailer. To fulfil the customer's requirement, order is sequentially placed across the CLSC entities considering the yield and quality of material at manufacturer and segregator stages respectively. The dealer collects the recyclable/disposable materials from the consumer and replenishes the stock of the segregator. The segregator performs the function of segregating/separating the recyclable materials from the materials received from the consumer and forwards it to the supplier stage. It is assumed that the dealer doesn't order for the recyclable materials from the source but only collects it from them. Hence, order variability at the dealer stage is not relevant and has not been accounted in the model.

The sequence of events in any period at any echelon is as follows: the order placed earlier is received, and the demand is fulfilled at the beginning of the period, the inventory level is reviewed, and ordering decision is made at the end of the period. A periodic review inventory policy

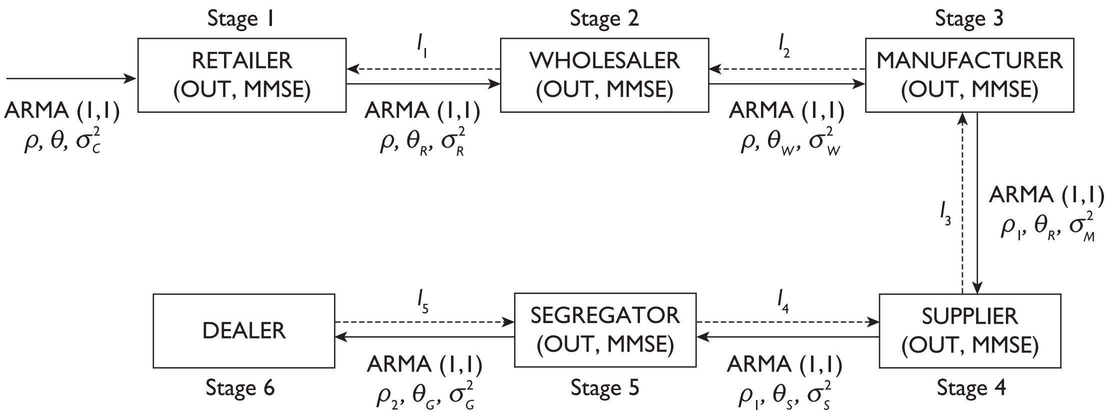

Figure 1. Structure of in CLSC showing demand at various stages with recycling as the only reprocessing options

is assumed with each echelon using OUT replenishment policy. A specific length of the review period has not been assumed hence the results will be valid for any unit of the review period adopted (day, week, month, etc.), as long as the unit are consistent throughout.

Retailer's Ordering Policy

Wholesalers anticipated (and retailer's order) quantity for period (t + 1) is:

$$R_{t+1} = \rho R_t + \lambda_1 \varepsilon_{t+1} - \eta_1 \varepsilon_t = \rho R_t + \varepsilon_{t+1,R} - \theta_R \varepsilon_{t,R}$$

Where, l_1 is the fixed lead-time for the wholesaler to fulfil retailer's order and the other parameters, ρ, θ, σ_ε^2 are as mentioned in section 2.

$$\theta_R = \eta_1/\lambda_1,\ \lambda_1 = (1+\rho\Omega_1 - \theta\Omega_1),$$
$$\eta_1 = (\theta + \rho\Omega_1 - \theta\Omega_1), \Omega_1 = (1-\rho^{l_1})/(1-\rho)$$

and

$$\varepsilon_{t,R} = \lambda_1 \varepsilon_t$$

Bullwhip ratio at the retailer stage =

$$(BW)_R = \frac{\text{Variance of relatiler stage order } (\sigma_R^2)}{\text{Variance of retail customer demand } (\sigma_C^2)}$$

Where, $\sigma_R^2 = \left[(1+\theta_R^2 - 2\rho\theta_R)/(1-\rho^2)\right]\sigma_{\varepsilon,R}^2$

and $\sigma_{\varepsilon,R}^2 = \lambda_1^2 \sigma_\varepsilon^2$

Thus, the demand process at the wholesaler stage from the retailer follows a scaled ARMA (1, 1) process with AR, MA parameters as ρ, θ_R respectively. The order variance at retailer stage and the error of the variance are given by σ_R^2, $\sigma_{\varepsilon,R}^2$ respectively.

Wholesaler's Ordering Policy

Manufacturer's anticipated (and wholesaler's order) quantity for period $(t + 1)$ is:

$$W_{t+1} = \rho W_t + (\lambda_1 + \rho\Omega_2\lambda_1 - \Omega_2\eta_1)\varepsilon_{t+1} - (\eta_1 + \rho\Omega_2\lambda_1 - \Omega_2\eta_1)\varepsilon_t = \rho W_t + \varepsilon_{t+1,W} - \theta_W \varepsilon_{t,W}$$

Where, l_2 is the fixed lead-time for the manufacturer to fulfil wholesaler's order and

$$\Omega_2 = (1-\rho^{l_2})/(1-\rho),$$
$$\theta_W = \alpha_1/\beta_1,$$
$$\alpha_1 = \eta_1 + \rho\Omega_2\lambda_1 - \Omega_2\eta_1,$$
$$\beta_1 = \lambda_1 + \rho\Omega_2\lambda_1 - \Omega_2\eta_1,$$
$$\varepsilon_{t,W} = \beta_1 \varepsilon_t.$$

Bullwhip ratio at the wholesaler stage =

$$(BW)_W = \frac{\text{Variance of wholesaler stage order } (\sigma_W^2)}{\text{Variance of retail customer demand } (\sigma_C^2)}$$

Where, $\sigma_W^2 = \left[(1+\theta_W^2 - 2\rho\theta_W)/(1-\rho^2)\right]\sigma_{\varepsilon,W}^2,$
$\sigma_{\varepsilon,W}^2 = \beta_1^2 \sigma_\varepsilon^2.$

Thus, the demand process at the manufacturer stage from the wholesaler follows a scaled ARMA (1, 1) process with AR, MA parameters as ρ, θ_W respectively. The order variance at wholesaler stage and the error of the variance are given by σ_W^2, $\sigma_{\varepsilon,W}^2$ respectively.

Manufacturer's Ordering Policy

Manufacturer converts the recyclable materials (with yield 'X') received from the supplier into the desirable single product. Suppliers anticipated (and manufacturer's order quantity) for period $(t + 1)$ is:

$$M_{t+1} = [\rho M_t + (\beta_1 + \rho\Omega_3\beta_1 - \Omega_3\alpha_1)\varepsilon_{t+1} - (\alpha_1 + \rho\Omega_3\beta_1 - \Omega_3\alpha_1)\varepsilon_t]/X = \rho_1 M_t + \varepsilon_{t+1,M} - \theta_M\varepsilon_{t,M}.$$

Where, l_3 is the fixed lead-time for the supplier stage to fulfil manufacturer's order and

$$\Omega_3 = (1-\rho^{l_3})/(1-\rho),$$
$$\theta_M = \gamma_1/\delta_1,$$
$$\gamma_1 = (\alpha_1 + \rho\Omega_3\beta_1 - \Omega_3\alpha_1)/X,$$
$$\delta_1 = (\beta_1 + \rho\Omega_3\beta_1 - \Omega_3\alpha_1)/X,$$
$$\varepsilon_{t,M} = \delta_1\varepsilon_t.$$

Bullwhip ratio at manufacturer stage =

$$(BW)_M = \frac{\text{Variance of manufacturer stage order } (\sigma_M^2)}{\text{Variance of retail customer demand } (\sigma_C^2)}$$

Where, $\sigma_M^2 = \left[(1+\theta_M^2 - 2\rho_1\theta_M)/(1-\rho_1^2)\right]\sigma_{\varepsilon,M}^2$

and $\sigma_{\varepsilon,W}^2 = \beta_1^2\sigma_\varepsilon^2, \rho_1 = \rho/X.$

Thus, the demand process at the supplier stage from the manufacturer stage follows a scaled ARMA (1, 1) process with AR, MA parameters as ρ_1, θ_M respectively. The order variance at manufacturer stage and the variance error are given by $\sigma_M^2, \sigma_{\varepsilon,M}^2$ respectively.

Supplier's Ordering Policy

Segregator's anticipated (and supplier stage order quantity) for period $(t + 1)$ is:

$$(Su)_{t+1} = \rho_1(Su)_t + (\delta_1 + \rho_1\Omega_4\delta_1 - \Omega_3\gamma_1)\varepsilon_{t+1} - (\gamma_1 + \rho_1\Omega_4\delta_1 - \Omega_3\gamma_1)\varepsilon_t = \rho_1(Su)_t + \varepsilon_{t+1,S} - \theta_S\varepsilon_{t,S}.$$

Where, l_4 is the fixed lead-time at the segregator stage to fulfil supplier's order and:

$$\Omega_4 = \left(1-\rho_1^{l_4}\right)/(1-\rho_1),$$
$$\theta_S = \Theta_1/\Xi_1,$$
$$\Xi_1 = \delta_1 + \rho_1\Omega_4\delta_1 - \Omega_4\gamma_1,$$
$$\Theta_1 = \gamma_1 + \rho_1\Omega_4\delta_1 - \Omega_4\gamma_1,$$
$$\varepsilon_{t,S} = \Xi_1\varepsilon_t.$$

Bullwhip ratio at supplier stage =

$$(BW)_S = \frac{\text{Variance of supplier stage order } (\sigma_S^2)}{\text{Variance of retail customer demand } (\sigma_C^2)}$$

Where, $\sigma_S^2 = \left[(1+\theta_S^2 - 2\rho_1\theta_S)/(1-\rho_1^2)\right]\sigma_{\varepsilon,S}^2$

and $\sigma_{\varepsilon,S}^2 = \Xi_1^2\sigma_\varepsilon^2.$

Thus, the demand process at the segregator stage from the supplier stage follows a scaled ARMA (1, 1) process with AR, MA parameters as ρ_1, θ_S respectively. The order variance at supplier stage and the variance error are given by $\sigma_S^2, \sigma_{\varepsilon,S}^2$ respectively.

Segregator's Ordering Policy

This is a critical stage in reverse supply chain for recycling activity. It is at this stage that the used recyclable materials (along with non-recyclable materials) received from the dealer is segregated, so that the recyclable materials could be sent to manufacturer for recycling. It is assumed that the mixture collected from the source by the dealer and then passed on to the segregator consists of a fixed proportion of non-recyclable materials $(1 - \psi)$; where ψ is proportion of recyclable materials from the total waste received for recycling.

Dealer's anticipated (and segregator stage order) quantity for period $(t + 1)$ is:

$$G_{t+1} = [\rho_1 G_t + (\Xi_1 + \rho_1\Omega_5\Xi_1 - \Omega_5\Theta_1)\varepsilon_{t+1} - (\Theta_1 + \rho_1\Omega_5\Xi_1 - \Omega_5\Theta_1)\varepsilon_t]/\psi = \rho_2 G_t + \varepsilon_{t+1,G} - \theta_G\varepsilon_{t,G}$$

Where, l_5 is fixed lead-time for the dealer stage to fulfil segregator stage order and

$$\Omega_5 = \left(1-\rho_1^{l_5}\right)/(1-\rho_1),$$
$$\rho_2 = \rho_1/\Psi, \theta_G = \chi_1/\varphi_1,$$
$$\varphi_1 = (\Xi_1 + \rho_1\Omega_5\Xi_1 - \Omega_5\Theta_1)/\Psi,$$
$$\chi_1 = (\Theta_1 + \rho_1\Omega_5\Xi_1 - \Omega_5\Theta_1)/\Psi,$$
$$\varepsilon_{t,G} = \varphi_1\varepsilon_t.$$

Bullwhip ratio at segregator stage =

$$(BW)_G = \frac{\text{Variance of segregator stage order } (\sigma_G^2)}{\text{Variance of retail customer demand } (\sigma_C^2)}$$

Where, $\sigma_G^2 = \left[(1+\theta_G^2 - 2\rho_2\theta_G)/(1-\rho_2^2)\right]\sigma_{\varepsilon,G}^2$

and $\sigma_{\varepsilon,G}^2 = \varphi_1^2\sigma_\varepsilon^2.$

Thus, the demand process for the dealer stage from the segregator stage follows a scaled ARMA (1, 1) process with AR, MA parameters as ρ_2, θ_G respectively. The order variance of segregator stage and the variance error are given by $\sigma_G^2, \sigma_{\varepsilon,G}^2$ respectively.

From the earlier statistical models (from the Section 'Retailer's Ordering Policy' to the Section 'Segregator's Ordering Policy '), it could be seen that when the retailer experiences ARMA (1, 1) demand process, then each of the higher echelons also face a scaled ARMA (1, 1) order processes. Furthermore, all the echelons can be expressed in terms of parameters of the customer demand process, that is, AR (ρ) and MA (θ) along with the lead-time. Therefore, all the echelons from retailer to dealers have complete information of the customer demand process with MMSE scheme. Figure 1 depicts the manner in which original ARMA (1, 1) demand process is changed by the OUT policy with the MMSE scheme as it proceeds upstream in CLSC for recycling.

Results Obtained from Model Analysis

This section attempts to perform sensitivity analysis on model parameters like ρ, θ, ψ and lead-time combinations on the performance of CLSC described in the Section 'Measure of Bullwhip Effect in Recycling Closed Loop Supply Chain (CLSC)'. The analysis is performed by considering various combinations of replenishment lead-times between successive echelons, with the sum of all downstream lead-times from retailer to dealer being fixed at 25 for the purpose of illustration.

The bullwhip ratio at each echelon has been calculated with two different sets of lead-times assuming that the market demand process parameters ρ and θ fall in range of $|\rho| < 1$ and $|\theta| < 1$ respectively. The two sets of lead-time combination considered are (7, 6, 5, 4, 3) and (3, 4, 5, 6, 7), where the numbers indicate the upstream replenishment lead-time between successive stages starting from the retailer stage. In subsequent discussions, the lead-time combination (7, 6, 5, 4, 3) and (3, 4, 5, 6, 7) are referred to as 'LT (a) set' and 'LT (b) set' respectively. Figures 2 (a and b) demonstrate the impact of parameters ρ and θ on the bullwhip ratio at each stage under LT (a) set, 100 per cent segregation of recyclable materials at the source (ψ = 100 per cent) and 100 per cent yield at manufacturer stage (X = 100 per cent). Similarly, Figures 3 (a and b) demonstrate the effect of parameters ρ and θ on the bullwhip ratio at each stage, with LT (b) set. In comparison to the present study, Hosoda and Disney (2006) attempted to examine only the impact of ρ on the bullwhip ratio. By comparing Figures 2 (a and b) and Figures 3 (a and b), the following could be concluded.

- An increase in ρ (AR parameter) increases the bullwhip effect, whereas increase in θ (MA parameter) decreases the bullwhip effect. The AR parameter significantly affects the bullwhip effect as compared to MA parameter.

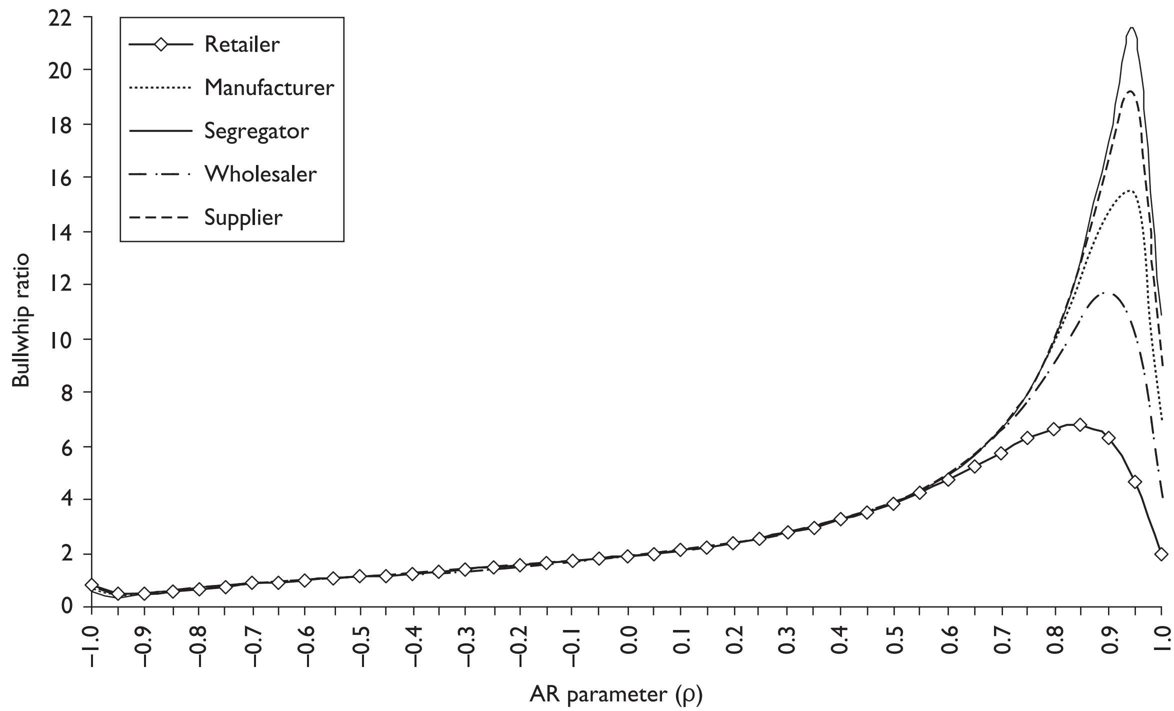

Figure 2(a). Effect of AR parameter (ρ) on bullwhip ratio of CLSC system, for $\theta = -0.60$ and with X = 100%, ψ = 100% $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$

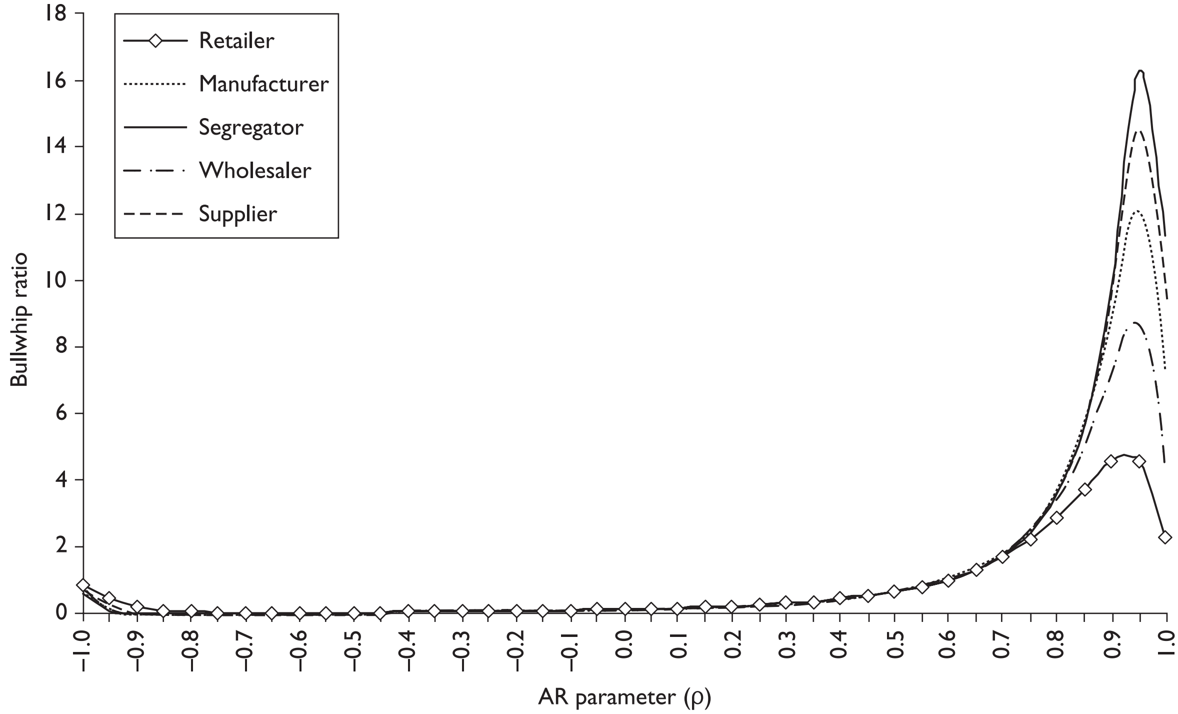

Figure 2(b). Effect of AR parameter (ρ) on bullwhip ratio of CLSC for $\theta = 0.60$ and with X = 100%, ψ = 100% $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$

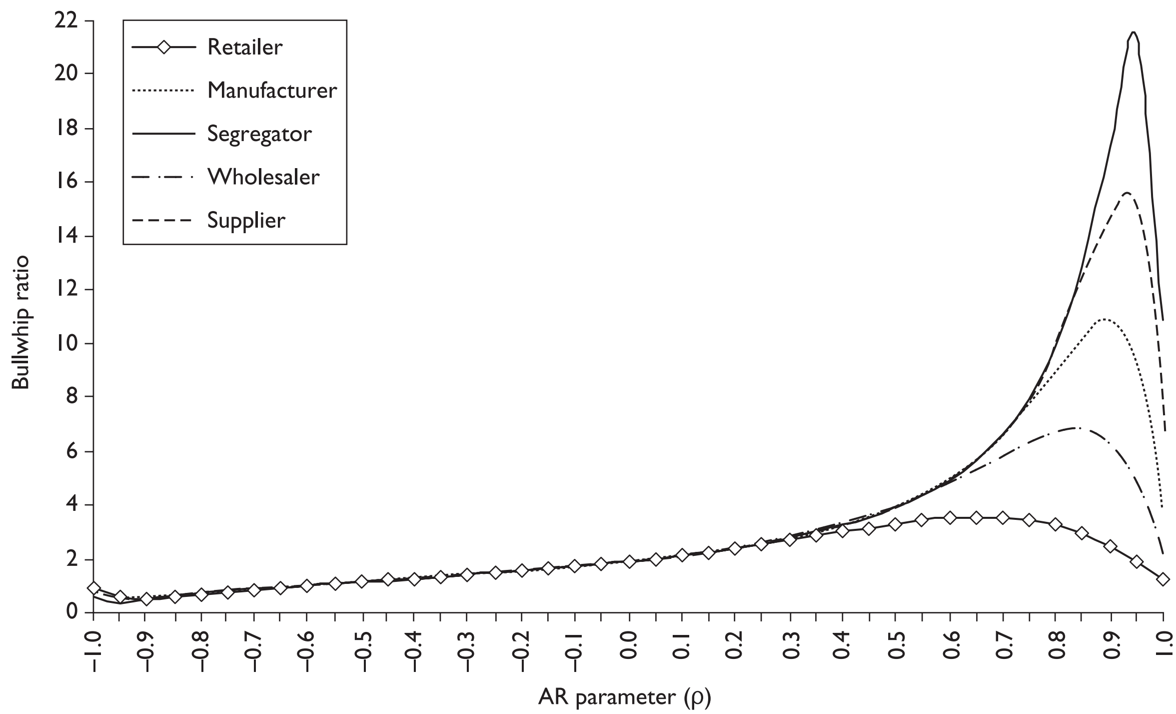

Figure 3(a). Effect of AR parameter (ρ) on bullwhip ratio of CLSC for $\theta = -0.60$ and with X = 100%, ψ = 100% $l_1 = 3$, $l_2 = 4$, $l_3 = 5$, $l_4 = 6$, $l_5 = 7$

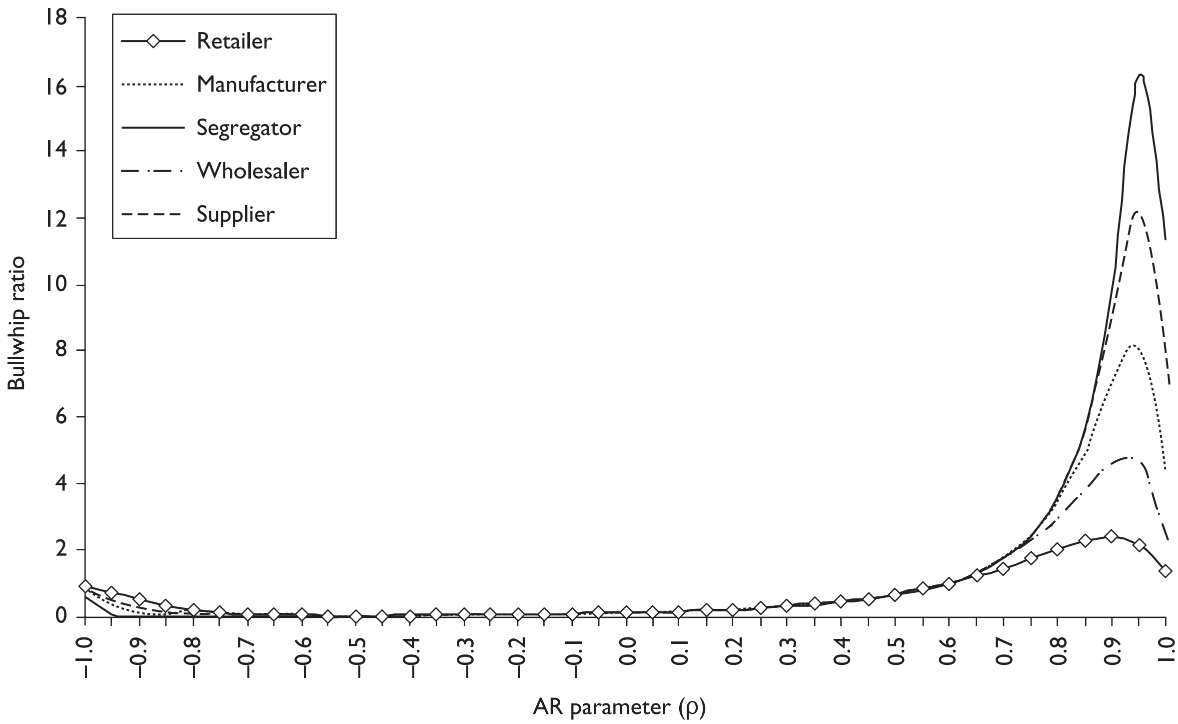

Figure 3(b). Effect of AR parameter (ρ) on bullwhip ratio of CLSC for $\theta = 0.60$ and with X = 100%, ψ = 100% $l_1 = 3$, $l_2 = 4$, $l_3 = 5$, $l_4 = 6$, $l_5 = 7$

- Bullwhip effect is eliminated at each echelon when the value of ρ matches the value of θ.
- Range of ρ for a similar bullwhip effect at each echelon increases with increase in θ.
- Range of ρ for a similar bullwhip effect is higher for LT (a) compared to LT (b) for a given θ.

For a given θ, the following observations are found to be similar to Hosoda and Disney (2006):

- Bullwhip ratio at segregator stage $[(BW)_G]$ is unaffected by lead-time combination between echelons and has the same shape under the constraint that the cumulative value of leadtime in the CLSC is constant (25 in this case).
- The magnitude of variance amplification at various stages is observed to be such that: $(BW)_R \leq (BW)_W \leq (BW)_M \leq (BW)_S \leq (BW)_G$ for both the lead-time combinations considered. This verifies that the variance amplification occurs as we move upstream in a CLSC also.

In real life practice, it is very difficult to attain $\psi = 100$ per cent and X = 100 per cent. Hence, Figures 4 (a and b) represent the effect of demand process parameters ρ and θ on bullwhip ratio for various stages of CLSC with lower degree of segregation (say $\psi = 95$ per cent) and yield of recyclable material (say X = 78 per cent) for given lead-time combination (say LT (a) set). Figures 4 (a and b) show the pattern at $\theta = -0.30$ and $\theta = 0.30$ respectively. The nature and shape of the figures are the same as discussed for X = 100 per cent and $\psi = 100$ per cent. It is observed that the bullwhip ratio increases significantly with the decrease in percentage yield at the manufacturer and degree of segregation at the source. Hence, an environmentally conscious decision maker would always invest in methodology and technologies that increase the segregation at the source, and consequently the quality of recyclable material (closer to 100 per cent) so as to reduce the bullwhip effect. This investment indirectly reduces the dependence on natural resource and leads to greening of ecosystem. Existence of waveform/unstable band range from $0.70 \leq \rho \leq 0.80$ for the manufacturer, supplier and segregator stage was also noticed in Figures 4 (a and b). The transition of AR parameter for an ARMA process from $\rho_1, \rho_2 \leq 1$ to $\rho_1, \rho_2 > 1$ for $0.70 \leq \rho \leq 0.80$ leads to waveform representing instability in bullwhip ratios at manufacturer (ρ_1), supplier (ρ_1) and segregator (ρ_2) stages. This violates the covariance

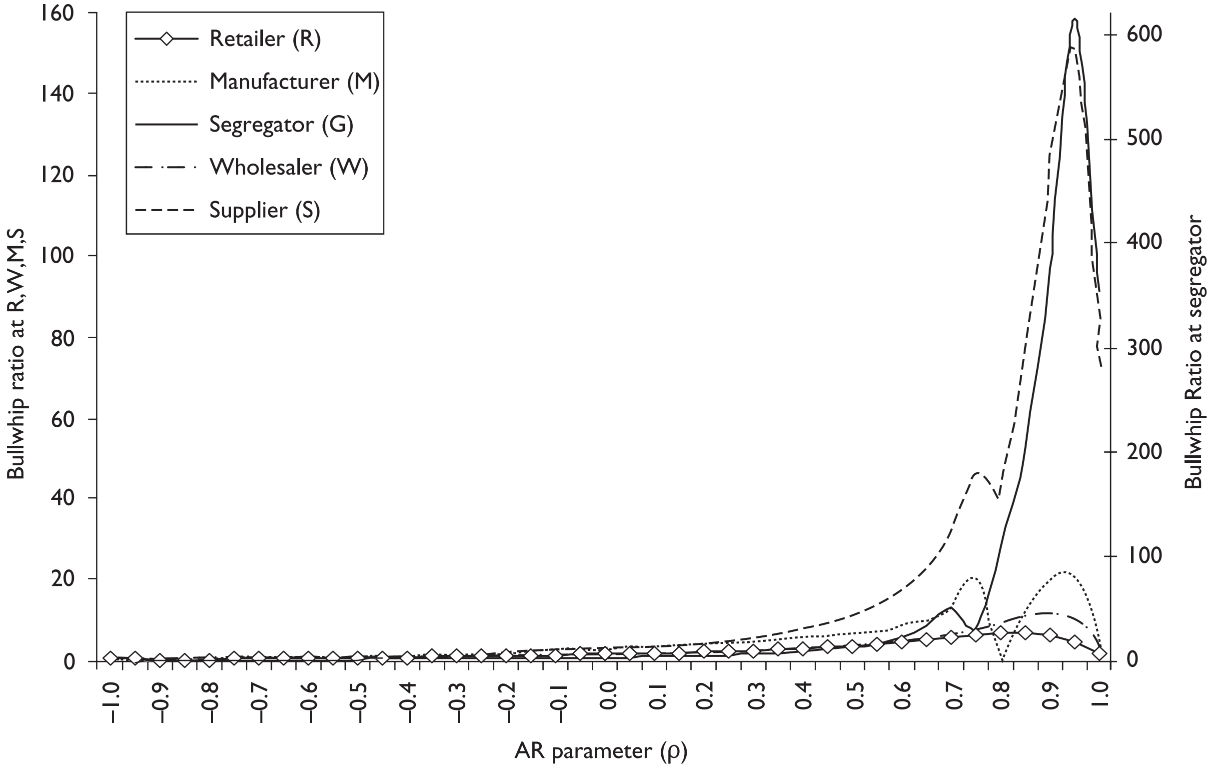

Figure 4(a). Effect of AR parameter (ρ) on bullwhip ratio of CLSC for $\theta = -0.30$ and with X = 78%, ψ = 95% $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$

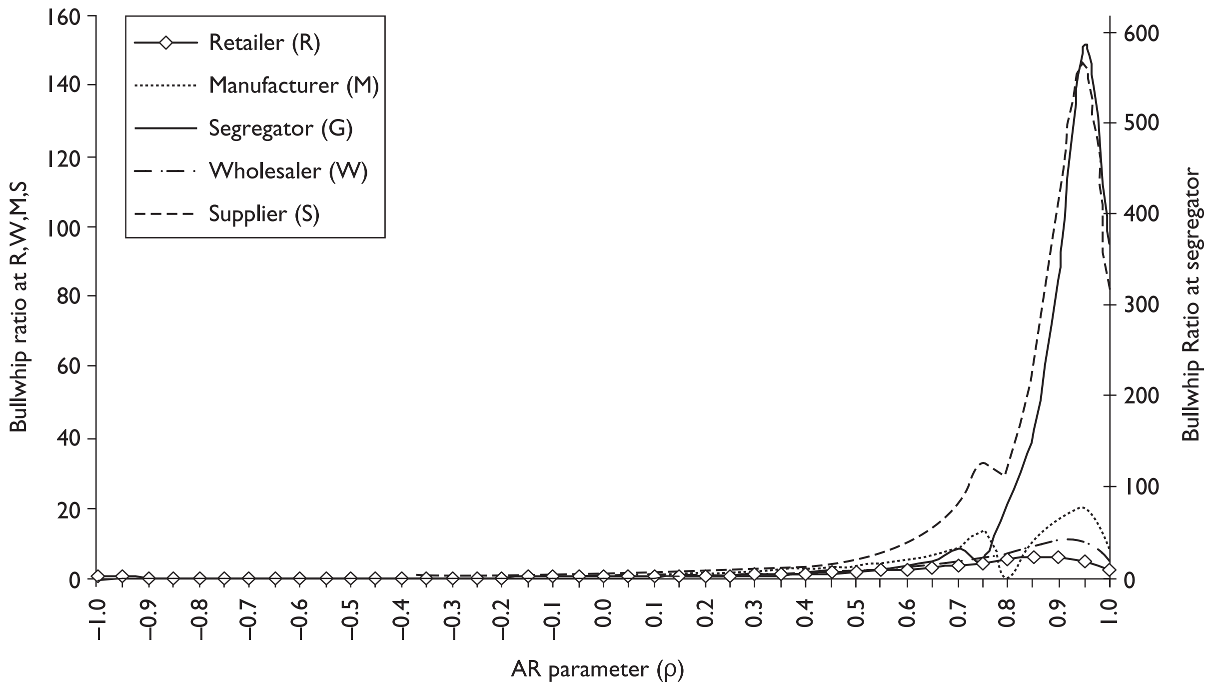

Figure 4(b). Effect of AR parameter (ρ) on bullwhip ratio of CLSC for $\theta = 0.30$ and with X = 78%, ψ = 95% $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$

stationarity condition (that is, $|\rho_1|$, $|\rho_2| < 1$) at manufacturer, supplier and segregator stages in the earlier mentioned range. Similar tendency was also observed for the other values of θ and lead-time combinations.

Figures 5 (a and b) depict the effects of decreased segregation at source (ψ) on the bullwhip effect in CLSC measured at the segregator stage for LT (a) set. The degree of segregation at the source is varied from 95 per cent to 60 per cent and its effect on the bullwhip ratio is observed, for X = 78 per cent with $\theta = -0.80$. The following inferences are made from the analysis:

- Variance amplification increases with decrease in ψ. The range of $-0.50 \leq \rho < -0.70$ and $0.50 \leq \rho \leq 0.70$ is an exception to this trend. In the earlier mentioned range there is a tendency of waveform or instability in the curve. For example, waveform of the curve for $\rho = 0.60$ is observed when degree of segregation at source is decreased from 85 per cent to 75 per cent. Similarly, curves for $\rho = 0.5, 0.7, -0.5, -0.6$ can be explained. The presence of waveform can be attributed to violation of covariance stationarity condition at segregator stage, that is, Auto Regressive parameter for an ARMA process changes from $\rho_2 \leq 1$ to $\rho_2 > 1$.
- With decrease in ψ, width of instable band range in terms of ρ increases.

To avoid instability in the system, stability analysis (Figures 6 [a and b]) was performed. The stability analysis determines the relationship among the degree of segregation at the source (ψ) and yield at the manufacturer (X) for different combination of AR parameter (ρ) and MA parameter (θ). For LT (a) set, Figures 6 (a and b) study the stability analysis at $\theta = -0.30$ and $\theta = 0.30$ respectively. Each point in Figures 6 (a and b) denote the minimum degree of segregation at source (ψ) for a given combination of ρ and X to avoid instability in the system. Further decrease in degree of segregation at source leads to an unstable system. The following inference could be made from Figures 6 (a and b):

- The presence of instability region is not affected by θ. It keeps the same shape for both the MA parameters considered. Similar, results were observed for other values of θ.
- Slope of curve increases with increase in ρ on both sides (i.e., $+\rho$ as well as $-\rho$).
- Unstable band range of ρ with varying degree of segregation at source can be determined for a given yield (X) at the manufacturer stage.

Figures 7 (a and b) examine the impact of different lead-time combinations on the bullwhip effect in CLSC with

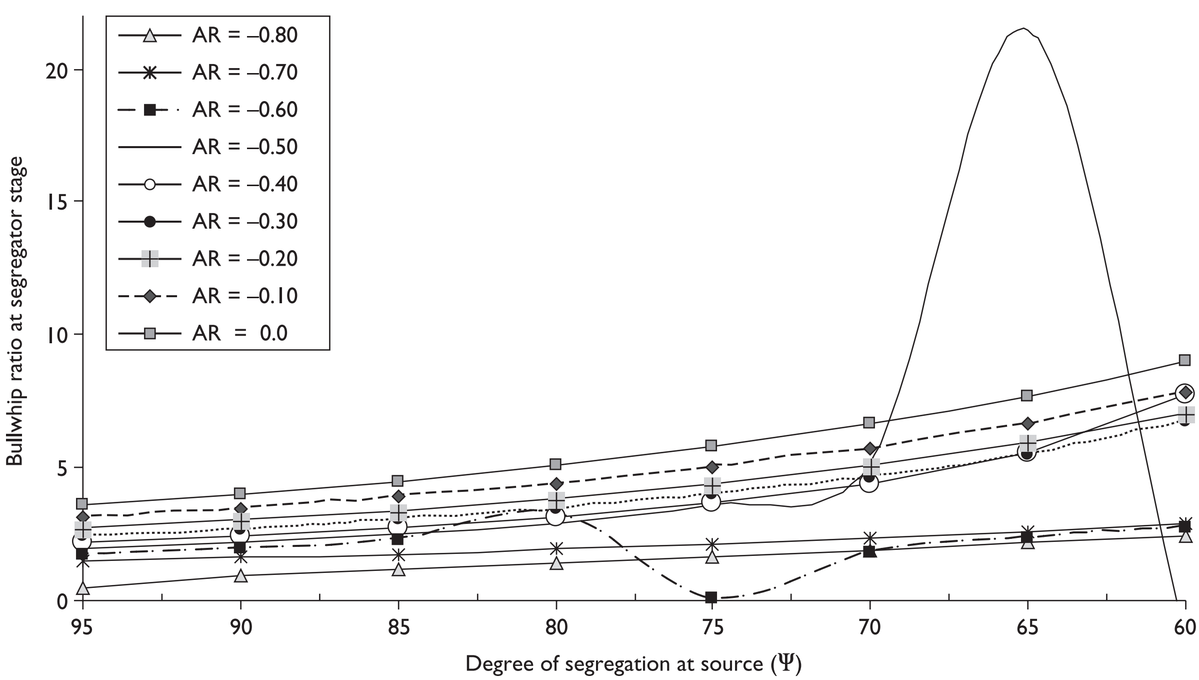

Figure 5(a). Effect of segregation at source (ψ) on bullwhip ratio for CLSC measured at segregator stage for $\rho \leq 0$, $\theta = -0.80$, X = 78%, Lead-time combination (7, 6, 5, 4, 3)

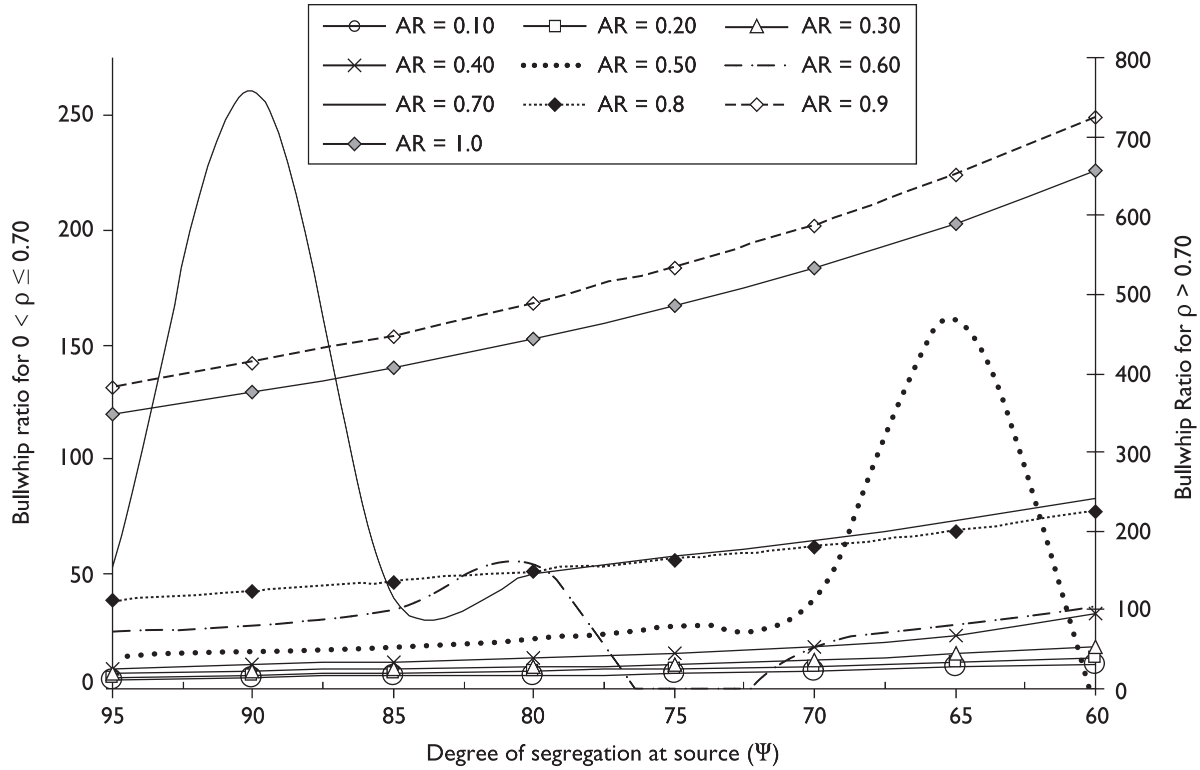

Figure 5(b). Effect of segregation at source (ψ) on bullwhip ratio for CLSC measured at segregator stage for $\rho > 0$, $\theta = -0.80$, X = 78%. Lead-time combination is (7, 6, 5, 4, 3)

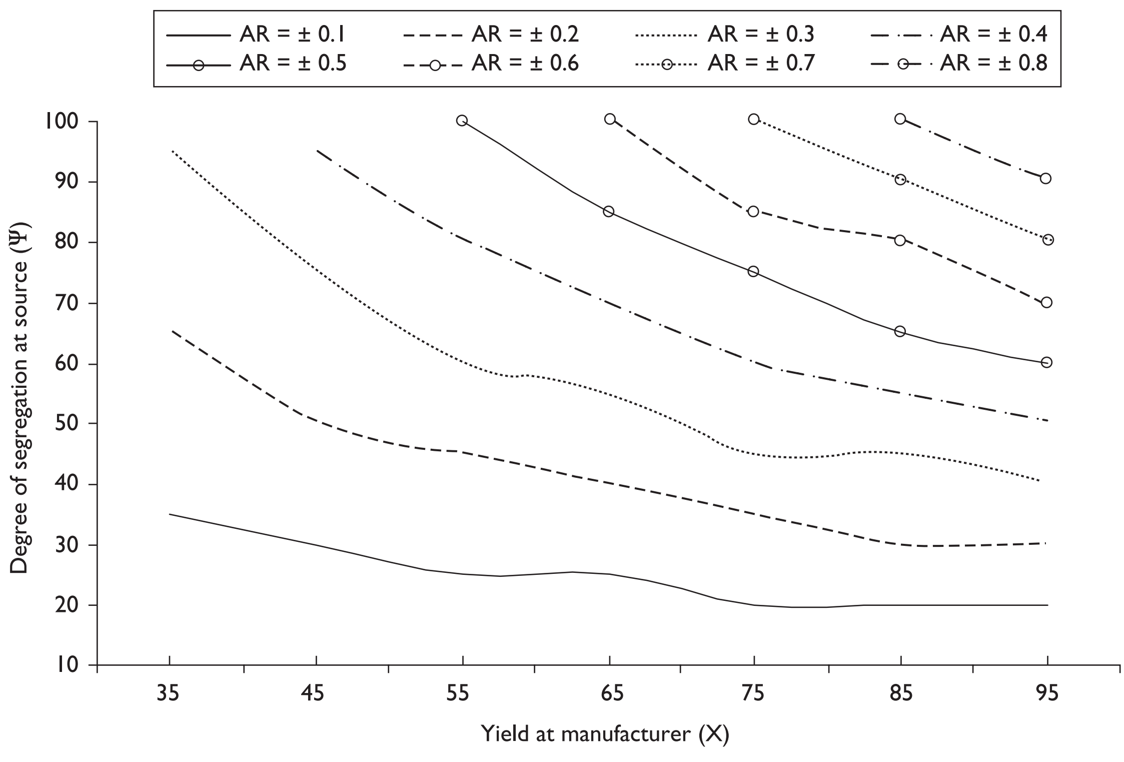

Figure 6(a). Stability analysis in Closed-Loop Supply Chain for lead-time combination (7, 6, 5, 4, 3), $\theta = -0.30$ and total CLSC replenishment leadtime is 25 unit

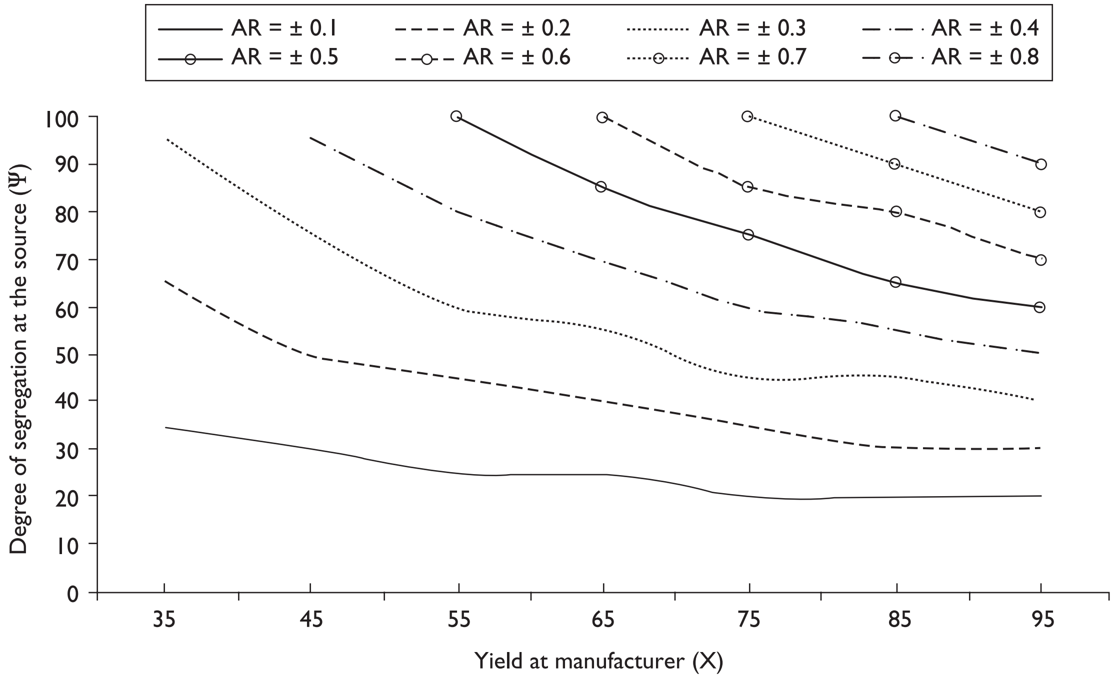

Figure 6(b). Stability analysis in Closed-Loop Supply Chain for lead-time combination (7, 6, 5, 4, 3), $\theta = 0.30$ and total CLSC replenishment leadtime is 25 unit

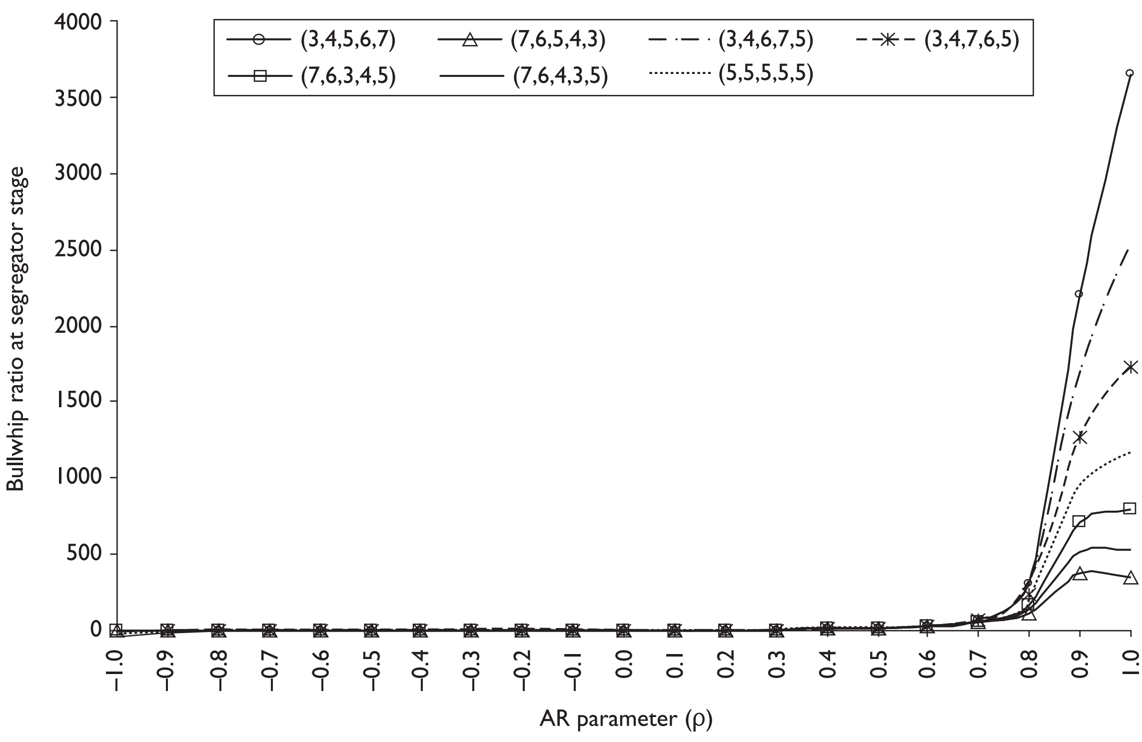

Figure 7(a). Effect of lead-time combination on bullwhip ratio of CLSC system measured at segregator stage

Notes: All combination $l_1 + l_2 + l_3 + l_4 + l_5 = 25$, X = 78%, $\psi = 95\%$, $\theta = -0.60$.
In combination (7,6,5,4,3) the inter-echelon lead-times are $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$

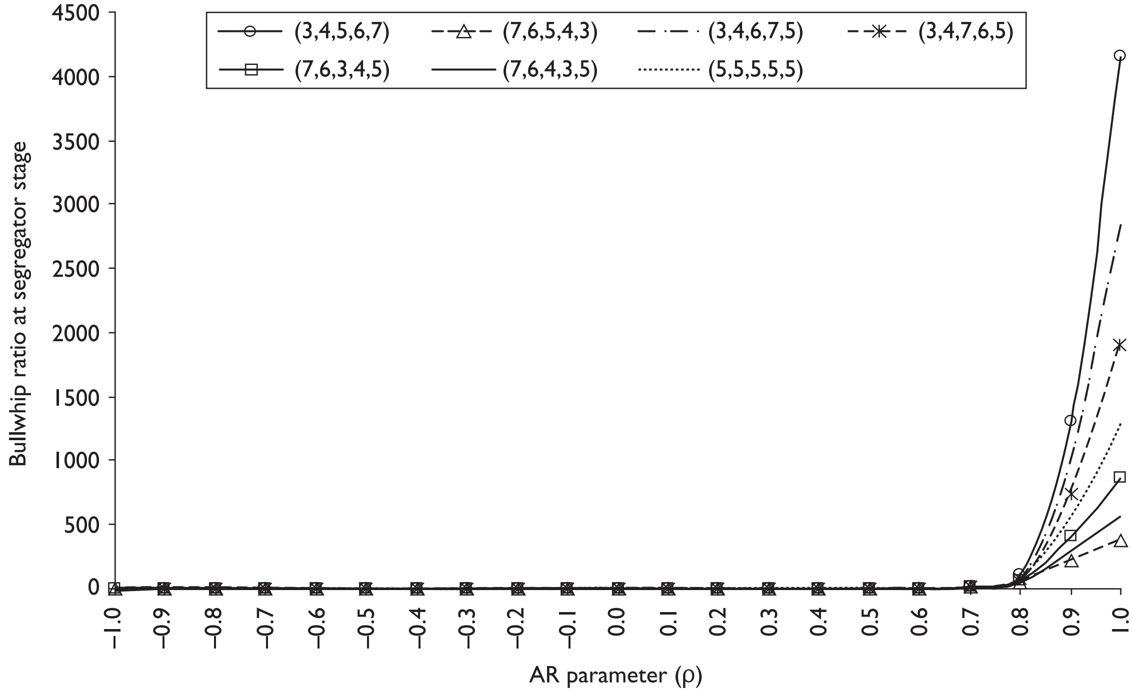

Figure 7(b). Effect of lead-time combination on bullwhip ratio of CLSC system measured at segregator stage

Notes: All combination $l_1 + l_2 + l_3 + l_4 + l_5 = 25$, and X = 78%, ψ = 95%, $\theta = 0.60$.
In combination (7,6,5,4,3) the inter-echelon lead-times are $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$.

recycling, for X = 78 per cent and ψ = 95 per cent and total lead-time across the CLSC fixed at 25 units. Figures 7 (a and b) represent the lead-time analysis at $\theta = -0.60$ and $\theta = 0.60$ respectively. Same trend was observed for other values of MA parameter (θ). The following observations could be made from the lead-time analysis:

- Bullwhip effect is similar for all lead-time combinations up to $\rho \leq 0.60$.
- For higher value of AR parameter ($\rho \geq 0.70$), the combination with lower lead-time between successive echelons closer to dealer stage of CLSC has lower variance amplification. Hence, while designing the CLSC the replenishment lead-time closer to retailer stage should be relatively higher than lead-time at echelons closer to dealer stage.

Figure 8 depicts variance amplification of customer demand when it passes from retailer to segregator stage for $\rho = 0.70$, $\theta = -0.70$, X = 78 per cent and ψ = 95 per cent for different lead-time combinations. It again shows that CLSC with lower replenishment lead-time closer to dealer stage and relatively higher lead-time closer to retailer results in lower bullwhip effect. Thus, the order of variance amplification for different lead-time combinations is:

$$(BW)_{7,6,5,4,3} < (BW)_{7,6,4,3,5} < (BW)_{7,6,3,4,5} < (BW)_{5,5,5,5,5}$$
$$< (BW)_{3,4,7,6,5} < (BW)_{3,4,6,7,5} < (BW)_{3,4,5,6,7}$$

Summary of Major Findings and Managerial Implications

This section attempts to provide useful insights and managerial implications from the proposed model to enable managers make appropriate policy decision to reduce bullwhip effect in CLSC. Some of the managerial implications of the model are:

- The ARMA (1, 1) demand process is transformed into a scaled ARMA (1, 1) process as it moves upstream in CLSC. The value of the ρ remains the same up to the wholesaler stage and then it gets changed at each echelon depending on X and ψ. However, the value of θ changes at each echelon

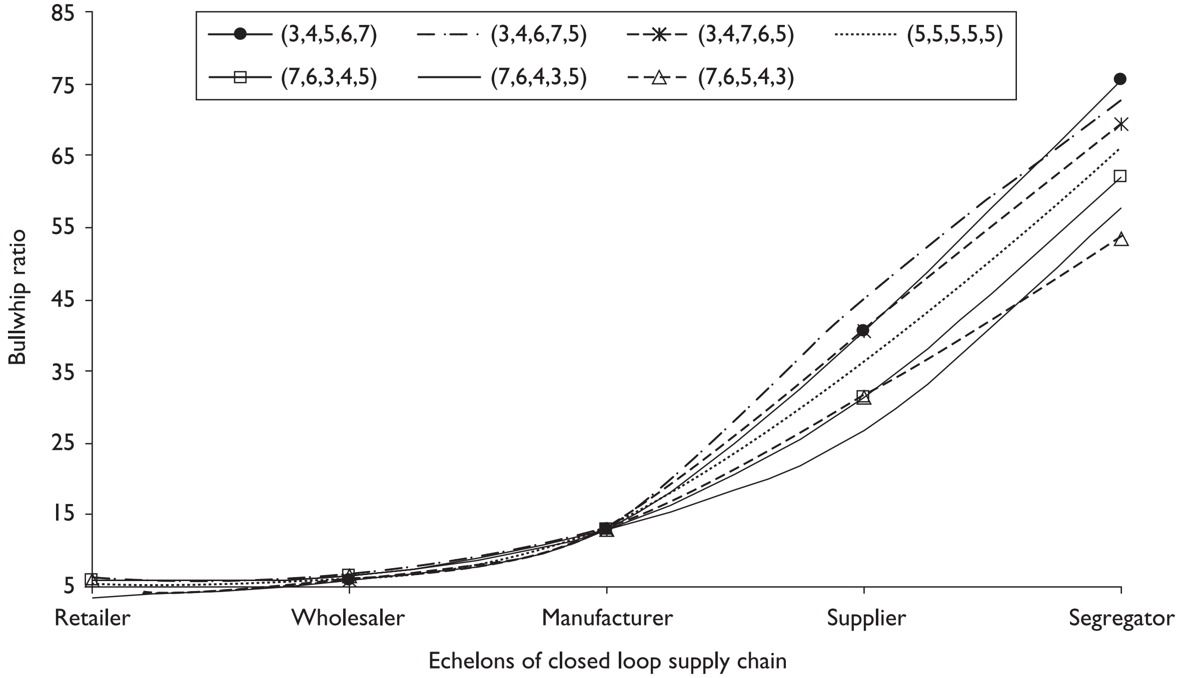

Figure 8. Effect of lead-time combination on bullwhip ratio at various echelons of CLSC system, for $\rho = 0.70$, $\theta = -0.70$, $X = 78\%$, $\psi = 95\%$

Notes: All combination $l_1 + l_2 + l_3 + l_4 + l_5 = 25$.
In combination (7, 6, 5, 4, 3) the inter-echelon lead-times are $l_1 = 7$, $l_2 = 6$, $l_3 = 5$, $l_4 = 4$, $l_5 = 3$.

and is a function of its replenishment lead-time, downstream lead-time, ρ, θ, X, ψ for the echelon. Thus, the ordering process contains complete information about the demand. With the knowledge of cumulative lead-time and the demand process, each supply chain partner is able to estimate the quantity of its demand over the lead-time.

- It was observed that bullwhip effect can be reduced by selecting lower value of ρ or higher value of θ under ARMA (1, 1) demand process. However, the bullwhip effect is eliminated when values of ρ matches the values of θ. This represents the ideal scenario to manage bullwhip effect.
- Bullwhip effect was found to increase with decrease in degree of segregation at source (ψ) as well as yield at manufacturer (X). With decrease in yield at manufacturer stage, there is an increase in demand for the recyclable material from the subsequent stages to meet the manufacturer is demand. The increased order size leads to higher variance amplification. Similar intuitive justification can be given for the effect of decrease in degree of segregation at source. Hence, the management must formulate policies for efficient collection and segregation at source as well as manufacturing strategies to enhance quality and yield. This compels the decision maker to go for environmentally conscious design through the use of better segregation techniques to get high yield recyclable materials.
- Lead-time analysis guides the CLSC partners in opting for a lead-time combination that results in the lowest possible bullwhip ratio. In an efficient CLSC design the replenishment lead-time should be allocated in the decreasing order starting from the highest for retailer to the lowest for the segregator stage. The intuitive justification of doing this is: at the later stages, larger quantities of materials are required to be transported in between the echelons (due to effect of X and ψ) hence lower lead-time will reduce the anticipated demand and orders at these stages.
- The stability analysis guides the management to carefully select the values of ρ so as to avoid any order process instability in CLSC for a given combination of X and ψ. Instability in CLSC due to lower yield at the manufacturer can be compensated by formulating strategies to increase the degree of segregation at source. It was observed that the presence of instability bandwidth is not affected significantly by the value of θ.

Concluding Remarks

A statistical model has been proposed in this article to investigate the bullwhip effect in a six echelon CLSC with recycling as the only reprocessing option. Although the bullwhip effect on forward supply chain has been extensively carried out, its behaviour in reverse supply chain and CLSC is scanty. This study attempts to fill the research gap by analyzing the impact of quality of recyclable materials and degree of segregation on demand variance amplification in CLSC. The retailer, wholesaler, manufacturer, supplier, segregator, and finally dealer are the stages of CLSC considered in the model. The retail customer demand for the recycled product is assumed to follow ARMA (1, 1) process and each supply chain participant adopts OUT replenishment policy with MMSE forecasting scheme. The finding of the study was similar to that obtained by Pati et al. (2010) under the similar setup but with AR (1) rail customer demand.

The proposed model can help the managers of CLSC in anticipating the downstream demand along with the quantitative measure of the bullwhip effect. Analysis based on the model suggests that bullwhip effect can be reduced by optimally selecting the market demand parameters. Lead-time analysis guides in formulating distribution strategies that minimize the bullwhip effect by assigning upstream members lower replenishment lead-time compared to downstream members. The article also acknowledges the facts that increase in segregation at the source as well as yield at manufacturer, decreases the bullwhip effect. This may motivate the decision maker to adopt environmentally conscious design approach by adopting the better segregation system closer to source and using recyclable materials for manufacturing.

The present study can be simplified by eliminating the supplier stage if after segregation at source, the used recyclable material is directly sent to the manufacturer. This rationalization will marginal reduce the bullwhip ratio at segregator stage without any loss of insights provided through the model developed in this article. The research work can be further extended in by quantifying the bullwhip effect in CLSC in terms of amplification ratios of the variance of net inventory levels. The impact of various other market demand processes like IMA (0, 1, 1), MA (0, 0, 1), ARIMA (1, 1, 1) etc. on the bullwhip effect in the CLSC is also worth investigating further.

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